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HYDRAULIC LABORATORY REPORT NO. 15

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CONCERNING THE MEASUREMENT OF VELOCITY AND
PRESSURE IN THREE-DIMENSIONAL FLOW
F. KRISAM
TRANSLATION FROM THE GERMAN

- - -

By

E. F. Wilsey

- - -

Denver, Colorado

1936

The Measurement of Velocity and Pressure
in Three-dimensional Flow

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A translation of:

Über die Messung von Geschwindigkeit und Druck in einer

dreidimensionalen Strömung

by

Dr. F. F. Krisam

in

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1936

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The Measurement of Velocity and Pressure in Three-dimensional Flow

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By Dr. Ing. F. Krisam, Karlsruhe, i. B.

(Communication from the Institute for Hydraulic Machinery of the Karlsruhe Technical University)

Summary

Both the magnitude and direction of the velocity of flow, and the hydraulic pressure can be measured with Von der Hegge Zijnen's five hole pitot sphere in a three-dimensional flow. Its operation and effective range will be discussed with respect to the findings of recent research on flow around a sphere. Experience obtained in applying the pitot sphere to practical problems will also be described.

1. General

For the exact measurement of three-dimensional flow, an instrument possessing both ease of manipulation and a high degree of accuracy is required. The familiar types of pitot tubes (Prandtl, Brabbee, etc.) are unsatisfactory for measuring, for example, flow around bends or behind centrifugal pumps. The new instrument appears well adapted to such measurements both as regards method as well as range of application.

Van der Hegge Zijnen has written at some length on the development of the pitot sphere.[1,2] ~~1,2.~~ His pitot sphere with five holes is an improvement on the earlier sphere with three holes invented by Borren.[3] ~~3.~~ Borren's pitot sphere developed in turn from the instruments of Cordier[4] and Ellon[5], was still very inconvenient and it possessed a narrow range of application. By increasing the number of holes to five Van der Hegge Zijnen has effected a substantial improvement.

2. Description and Application of the Pitot Sphere

The instrument consists of the sphere, a, (see figure 1), the shaft, b, and the graduated circular plate, c. The five holes are arranged on two meridians normal to each other. The graduated circular plate is so fixed with respect to the shaft that its zero lies in the same plane as holes 1, 2 and 3. Holes 4 and 5 lie equidistant from hole 2.

When in use, the instrument is rotated about the shaft until manometers 4 and 5 indicate the same reading. Then the plane passing through holes 1, 2 and 3 lies in the direction of the stream flow. The angle α between the meridian plane 1, 2 and 3 and a fixed reference plane can be read on the protractor plate.

Let h_1, h_2, h_3, etc. represent the head on the manometers corresponding to each hole, then:

$$h_1 = p/\gamma + k_1 c^2/2g$$
$$h_2 = p/\gamma + k_2 c^2/2g$$
$$\vdots$$
$$h_5 = p/\gamma + k_5 c^2/2g$$

in which

p/γ = the pressure head
c = the magnitude of the velocity
k_1, k_2, etc. = the coefficients for the holes

The coefficients will be assumed to be independent of the magnitude of the velocity and dependent only on the angle δ, that is on the angle between the direction of the flow and the axis of hole 2. Because of this dependance, it must be possible to determine angle δ from the pressure readings. Knowing the coefficients we obtain

$$h_1 - h_2 = \frac{c^2}{2g}(k_1 - k_2)$$

$$c^2/2g = \frac{h_1 - h_2}{k_1 - k_2} = \frac{h_2 - h_3}{k_2 - k_3} = \frac{h_2 - h_4}{k_2 - k_4} = \cdots$$

$$p/ \quad = h_1 - k_1 c^2/2g = h_2 - k_2 c^2/2g = \cdots$$

As regards the coefficients, the applicability of the pitot sphere is conditioned on:

a. Whether the coefficients can be determined with sufficient accuracy.

b. Whether an accurate value of δ can be obtained since the various coefficients depend on δ.

c. Whether the coefficients are independent of the magnitude of the velocity throughout a sufficiently wide range.

d. Whether the pressure on the surface of the sphere is symmetrically distributed with respect to the meridian passing through holes 1, 2 and 3, thus making it possible to determine the angle α since then equal pressures exist at holes 4 and 5.

a. The coefficients may be determined either experimentally or theoretically. The theoretical calculation of the coefficients is possible only under ideal conditions (ideal fluid). Therefore the only reliable values are those found by experiment. The pressure distribution on a submerged body can be calculated by using the potential theory for a frictionless fluid. This calculation happens to be extremely simple for the case of the pitot sphere (neglecting the influence of the shaft). The velocity potential for parallel flow of velocity c_∞ around a sphere[6] and also for the flow due to a double source is:

$$\phi = c_\infty x\left(1 + \frac{R^3}{2r^3}\right)$$

The velocity components are obtained by a differentiation and the total velocity at any point from $c^2 = c_x^2 + c_y^2 + c_z^2$. We are concerned only with the velocity at the surface of the sphere where r = R.

$$\text{Thus} \quad c_o^2 = c_\infty^2 \, 9/4 \left(1 - \frac{x^2}{r^2}\right)$$

By Bernoulli's equation, the pressure head is:

$$\frac{p}{\gamma_o} = \frac{p}{\gamma_\infty} + \frac{c_\infty^2}{2g}\left(\frac{9}{4}\ \frac{x_0^2}{r^2} - \frac{5}{4}\right)$$

The expression in the parenthesis is identical with the coefficient k.

Since $\cos \delta' = \frac{x}{r}$ (see figure 2), we have

$$k = 1/4\,(9 \cos^2 \delta' - 5)$$

When $\delta' = 0$, k = 1 (maximum value). The position of the inflection point of pressure is given by:

$$\cos \delta' = \sqrt{5/9}, \text{ or } \delta' = 42°$$

The resistance of the sphere to flow cannot be computed theoretically because the pressure distribution on the back half of the sphere is identical with that on the front half. In consequence of the discontinuity of the boundary layer of an actual fluid the pressure on the back of the sphere develops only partially and a decided resistance results. The true pressure distribution on the front half of the sphere is in almost complete agreement with the theoretical distribution because the frictional influence is confined to a very thin layer. Figure 3 shows the theoretical curve as well as several experimental k- curves. The calibration curve of hole 1 was chosen for comparison because there the disturbing influence of the shaft was least. The instrument should be calibrated in spite of the good agreement shown in figure 3 because of this influence of the shaft. Figure 4 shows a mounting for the pitot sphere for calibration in a long straight conduit such as was used in the Karlsruhe Institute for Hydraulic Machinery. The pitot sphere was calibrated at at the middle of a straight length of pipe 12,000 mm long and 190 mm in diameter. An ordinary Prandtl pitot tube is placed in a similar position for comparison. The pitot tube and pitot sphere are placed at equal distances from the axis of the pipe (about 55 mm), the symmetry of the velocity profiles at the traverse having been established by measurements. All calibrations were performed in water. The mounting of the pitot sphere permits the rotating of the whole instrument in order to adjust for any angle of flow, , and also the turning of the shaft so that the reading of manometer [illegible] be set equal to the reading of manometer 2, Figure 5 shows a calibrated curve. The shape of curve, k_4, follows in consequence of the location of hole 4. Holes 4 and 5 lie at about 51° on either side of hole 2. When the angle of flow for hole 2 is zero ($\delta = 0$) the angle of flow of hole 4 is 51° and will increase as δ increases. Hole 4 therefore lies always in a region of low pressure. For a theoretical calculation only the angle of flow, δ, is needed. Since a large error is to be expected as a result of the large angle of flow, this calculation is omitted. An additional pressure arises due to the shaft and is greatest at holes 3 and 2 which lie nearest to the shaft. The whole pressure field is shifted in the direction of $+\delta$ on account of the shaft. Figure 6 shows the arrangement of the manometers for the calibration. Water can flow through the manometer connections in both directions. Either water or acetylene tetrabromide (specific gravity = 2.98) may be used for the manometer fluid depending on the size of reading desired. Convenient and accurate work is possible with a special manometer reading device. The calibration should be made throughout a range of δ from +60 degrees to − 60 degrees advancing by 5 degrees at a time as often as is necessary to eliminate accidental errors of observation. The discussion of the accuracy of the calibration will be continued later.

b. The coefficient for a single hole is calculated from the values, p/γ and $c^2/2g$ as measured by an ordinary pitot tube.

Thus:

$$k_1 = 2g/c^2 \, (h_1 - p/\gamma)$$

$$k_2 = 2g/c^2 \, (h_2 - p/\gamma)$$

$$\vdots$$

$$k_5 = 2g/c^2 \, (h_5 - p/\gamma)$$

Figure 5 shows the curves for the coefficients. A ratio must be found in terms of the manometer readings h_1, h_2, etc., so that the angle, δ, can be computed. $\dfrac{k_5 - k_1}{k_2 - k_4}$ is such a ratio and will be noted by k_δ. k_δ is uniquely and always dependent on δ especially in the range from approximately +60 degrees to −60 degrees. It is further necessary to express k_δ in terms of the manometer readings since k_1, k_2 etc. are computed from them.

Thus: $$k_\delta = \frac{k_5 - k_1}{k_2 - k_4} = \frac{h_5 - h_1}{h_2 - h_4}$$

The chief advantage of the pitot sphere lies in this unique and simple relation. It is easily seen that if one of the holes 1 to 4 is eliminated, the value of the sphere for the unique determination would be sharply limited.

c. The utility of the pitot sphere is dependent on the validity of the fundamental assumption that the coefficients are independent of the magnitude of the velocity. The resistance curve of a sphere sheds light on this question (see figure 7). The resistance of a sphere depends on the pressure distribution. It is assumed that the pressure distribution remains constant if the resistance of the sphere remains constant at various velocities. The resistance coefficient is practically constant throughout a range of Reynold's numbers from $R = 2 \times 10^4$ to 1.5×10^5 (see figure 7). The limits correspond to an air velocity from 26 to 200 meters per sec. and a water velocity from 2 to 16 meters per second for a sphere 12 mm. in diameter. The upper velocity limit is seldom exceeded in contrast to the lower limit which seems somewhat high. The variation of the resistance coefficient between $R = 7 \times 10^4$ and $R = 2 \times 10^3$ however, is so small that the lower limit may be assumed to lie below 1 meter per second for water and 13 meters per second for air for the given sphere diameter. The fundamental assumption has been verified by a series of pitot sphere calibrations and pressure measurements at various values of Reynold's number. The following table gives a summary of several experiments known to the author:

Experiment by	Medium	Sphere mm.	Velocity m./sec. Min.	Velocity m./sec. Max	Reynold's Numbers R min.	Reynold's Numbers R max.
1, 2, 3 Meyer	Air	10	6	30	3.8×10^3	1.9×10^4
v. d. Hegge Zijnen	"	12	—	20	—	1.6×10^4
Ermisch 10	Water	20	0.05	—	7.47×10^2	—
"	"	38.2	0.477	0.98	1.75×10^4	2.81×10^4
Karlsruhe Institute	"	12	3.0	4.5	2.9×10^4	4.4×10^4
	"	12	1.0	6.6	9.6×10^3	6.5×10^4
	"	5	1.0	5.5	4×10^3	2.2×10^4

Meyer's experiments as well as those of the Karlsruhe Institute show no measurable variation of the resistance coefficient although they were performed throughout a wide range of Reynold's numbers. Ermisch used the lowest velocity with his 20 mm. sphere. He experimented with both spheres in a channel of only

150 mm. width so that the influence of the walls ought to affect his results. In spite of this, his pressure distribution curves show remarkable agreement even for so large an angle of flow as approximately 70 degrees. The first deviations began at an even greater value. It is concluded from this that the dropping off of the pressure due to the vortex region, where accurate measurements are difficult, lies at an angle of more than 70 degrees. Meyer's measurements also show large deviations at 70 degrees. The Karlsruhe experiments show disturbances also at greater angles. However, the coefficients were in no way dependent on the velocity. The results of repeated experiments with constant velocity give the same percentage deviation from one another as the results from experiments with different velocities. Measurements in the above range were probably subject to major difficulties. The differences between individual measurements however, were small enough so average values of the coefficients could be obtained even for different observers and different series of tests. On the basis of these experiments it is concluded that the coefficients are constant throughout a range of Reynold's numbers from 3.5×10^5 to over 1×10^6.

d. Holes 4 and 5 are equi-distant from hole 2. Angle α is determined when their pressure readings are equal (see figure 1). In order to obtain the maximum sensitivity, holes 4 and 5 should be located at points of the sphere where the pressure gradient is a maximum. Van der Hegge Zijnen has found by experiment that the slope of the pressure curve is greatest at an angle of about 50 degrees and he placed holes 4 and 5 accordingly. Only a symmetrical pressure distribution on the pitot sphere will justify its use as a measuring instrument.

Krell[11] has measured the pressure distribution at various velocities and found the pressure distribution to be considerably unsymmetrical. His measurements are not to be relied on. An unsymmetrical and unstable pressure distribution will be met with only in that range of Reynold's numbers where the resistance coefficient falls off sharply. The acceptance of a symmetrical pressure ~~sure~~ distribution, at least in the uncritical range, is founded on the findings of various other experimenters. The experiments of Ermisch show a beautifully symmetrical pressure distribution in the stable range for both a sphere and a cylinder. The result obtained on the cylinder is especially noteworthy since here due to the formation of the Karman vortex tract symmetrical pressure distribution should be expected to occur much earlier. Eisner's[12] measurements are also interesting in this connection. As a result of his experiments on a circular cylinder, he concludes: the pressure distribution is symmetrical in the stable range; it is greatly unsymmetrical in the critical range (sudden dropping off of the resistance coefficient); strong lateral motion occurring here (analogous to Krell's tests with c = 50 m. / sec); in the range above the critical point, however, in which the pressure distribution is entirely different from that in the range below the critical point it is essentially symmetrical again. Eisner measured the pressure on both halves of the cylinder simultaneously.

Finally, an unsymmetrical and unstable pressure distribution was not observed in the numerous experiments at the Karlsruhe Institute either for water or air.

4. Sources of Error in Measurements with the Pitot Sphere

It has previously been pointed out that the pitot sphere is primarily adapted to measurements in three-dimensional flow. Its operation is relatively simple and it can be used for the measurement of magnitude as well as direction of the velocity.

The first objection to the pitot sphere is that the measurement is not made at a point but on a circular area of relatively greater magnitude. This objection is especially valid if the channel is relatively small. The manufacturer* cannot make a sphere smaller than about 12 mm. without considerable difficulty. This size is not objectionable for wind tunnel measurements, but in hydraulic experiments where smaller cross sections are frequently dealt with, the effect is greater. The pitot sphere shares this difficulty, more or less, with the ordinary pitot tube. The difference between the parallel flow with equal velocity distribution and known degree of turbulence in a calibration and the wholly arbitrary flow found in experiments falls with greater weight on the pitot sphere. A varying degree of turbulence in calibration and experimental set-ups is an inevitable source of error for all pressure measuring instruments. The pitot sphere must be calibrated in uniform parallel flow. When the pitot sphere is applied to flow which has a large pressure or velocity gradient $\partial p/\partial y$ and $\partial c/\partial y$ respectively, additional errors arise which can only be analysed i n certain special cases. Finally, the influence of the nearness walls on the coefficients is to be considered. In general, the coefficients of the holes on the half of the sphere nearer the wall become smaller. For example for the position shown in figure 8, k_3 is affected the most and k_2 and k_4 less. The changes in the last two will be proportionally about equal, so that the difference $k_2 - k_4$ will not be affected much. $c^2/2g$ is computed from $\frac{h_2 - h_4}{k_2 - k_4}$. Therefore errors arising from the proximity of the wall as a rule will not be very large. For calculating the pressure for the case shown in figure 8, hole 1 is the most expedient to use.

this is a partial derivative symbol

Thus $$p/\gamma = h_1 - k_1 \cdot c^2/2g$$

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*R. Fuess, Berlin, has informed me that he can supply an 8 mm. sphere at an increase in price.

If the other side of the sphere is near the wall, an analysis similar to the above should be made. Summing up, the chief sources of error are:

1. Measurement on a relatively large circular area instead of at a point.

2. Different degrees of turbulence in calibration and actual test set-ups.

3. Calibration in parallel flow with a uniform velocity distribution; use in flow with non-uniform velocity distribution.

4. Alteration of the coefficients in measurements near a wall.

The majority of these sources of error apply, however, to all sorts of pressure measuring instruments.

5. Measurements Using a Pitot Sphere

Excellent results can be obtained with the pitot sphere despite these sources of errors as experiments at the Karlsruhe Institute prove. Figure 9 shows the results of measurements in the draft tube of a Kaplan turbine. The turbine runner was removed in this experiment and a number of radial guide vanes were installed in place of the guide vane apparatus. Thus true meridional flow was produced. Two possible checks on the accuracy of the experiment were available. First, the discharge calculated from the velocity profile must agree with the discharge measured by an ov[e]rfall weir. Second, the total energy at the place of measurement must be approximately equal to the total fall, since only small friction losses occur up to this point.

The velocity components are found from the radial position of the pitot sphere (see figure 1).

Axial component, $c_a = c \cos\delta \sin\alpha$
Tangential ", $c_u = c \cos\delta \cos\alpha$
Radial ", $c_r = c \sin\delta$

The discharge calculated from $Q = 2\pi \int r\, c_a\, dr$ gave 235.5 liters per second whereas the weir measured 228 liters per second.

The error of about 2 percent is small considering the size of the spherical pitot tube (12 mm. diameter with a measuring range of 87 mm.) The drop in head amounted to 1 meter so that the total power, $QH = 228$ meters - kilograms/second. The total power calculated from the measurement was:

$$E = 2\pi\gamma\,(p/\gamma + h + c^2/2g)\; r\, c_a\, dr = 233.5 \text{ meter-kilograms/sec.}$$

whereas an $E = 228\,(1.02)^3$ ought to be expected, since c_a is already in error by 2 percent and this error enters to the third power in the formula. The difference of about 3.5 percent between the measured and calculated value of the energy is due in part to the friction and in part to the error in the determination of the pressure. The profile shows that in spite of the guide vanes, a small c_u component is present. As a result of the slope of the boundary walls, a positive c_r component exists on the outside and a negative value at the runner hub. The retardation at the runner hub because of the increasing cross-section is conspicuous in the pressure increase as well as in the dropping off of the velocity profile.

The flow conditions in figure 9 were still very uniform and could have been measured with an ordinary pitot tube without serious error. The second example is as shown in figure 10.* In this case the discharge from a Kaplan turbine with partial gate opening is measured. The measurement was made at the same place as in the experiment in figure 9. The experimental discharge was in error by only 0.8%. Thus, notwithstanding the large variations in velocity along the traverse, very satisfactory results were obtained. Both examples are taken from a large series of experiments with very diverse flow conditions [13] and the results are therefore not accidental. Such an experimental accuracy indicates that the various possible errors more or less cancel each other.

The possibility of checking the velocity and also the pressure by computing them from different ratios, for example,

$$c^2/2g = \frac{h_1 - h_2}{k_1 - k_2} = \frac{h_2 - h_3}{k_2 - k_3} = \ldots\ldots\ldots$$

is pointed out.

In general, when calculating the velocity and pressure, the data obtained from the holes which lie outside the vortex region should be used.

*The curves in figures 9 and 10 are computed on the basis of a 1 meter head. The actual head in the experiment was 4.8 meters.

Bibliography

1. Van der Hegge Zijnen, "Improved Instrument for the Simultaneous Determination of the Static Pressure, the Magnitude and the Direction of the Velocity of Fluids." Proc. Roy. Akad. Amsterdam, Vol. XXXII, No. 5, 1929.

2. Van der Hegge Zijnen, "Verbetered instrument ter bepaling van den statischen druk, de grotte en richting der strommelheid van vloeistoffen." De Ingenieur 44, Nr. 35, 1929.

3. H. Meyer, "Onderzoek betreffende de meetmethode met de Pitotbuis van dr. ir. J. J. Boren." De Ingenieur 43, Nr. 27, 1928.

4. W. v. Cordier, "Stromungsuntersuchungen an einem Rohrkrummer." (Investigation of Flow in a curved pipe) Dissertation, Berlin, 1916.

5. Ellon, Mitt. Forschungsarbeiten VDI, Heft 102.

6. Foppl. Techn. Mechanik, Band IV.

7. Krisam-Lorenz, "Messtechnisches aus dem Institut fur Stromungsmaschinen der Technischen Hochschule Karlsruhe." (Measuring technique from the Institute for Hydraulic Machinery of the Karlsruhe Technical University) Die Messtechnik, 1930.

8. Wieselsberger, "Uber den Luftwiderstand von Kugeln." (The air resistance of Spheres) Zeitschrift fur Flugtechnik und Motor luftschiffahrt, 1914

9. Hutte, Band I, 25 Aufl. S. 374.

10. Abhandlungen aus dem Aerodynamischen Institut der Technischen Hochschule Aachen. Heft 6.

11. Krell, "Druckverteilung an der luftumstromten Kugel." (Pressure Distribution on a Sphere in an Air Stream) Z FM 1931, Heft 4.

12. Eisner, "Widerstandsmessungen an umstromten Zylindern." (Resistance Measurement on the Flow around Cylinders), Mitt, Preuss, Versuchsanstalt fur Wasserbau und Schiffbau, Berlin, Heft 4, 1929.

13. Krisam: "Untersuchung einer neuen Saugrohrform fur Turbinen-Schnellaufer." (Investigation of a New Draft Tube for High Speed Turbines), Mitteilungen des Instituts fur Stromungsmaschinen der Technischen Hochschule Karlsruhe, Heft 2, 1932.

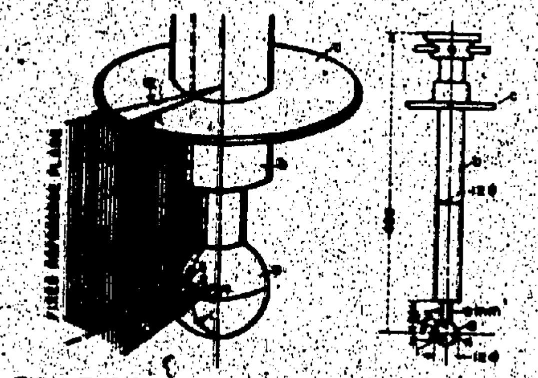

FIG. 1 THE PITOT SPHERE OF VAN DER HEGGE ZIJNEN

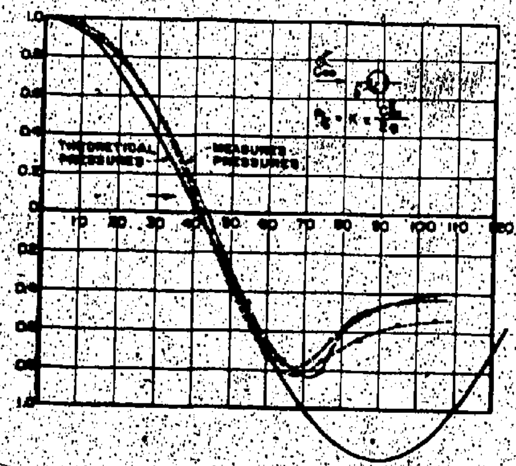

FIG. 3 THEORETICAL AND ACTUAL PRESSURE DISTRIBUTION ON THE SPHERE

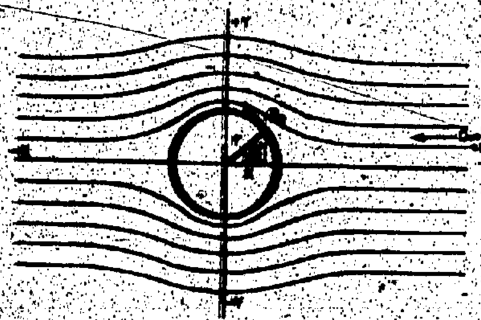

FIG. 2 FLOW AROUND A SPHERE

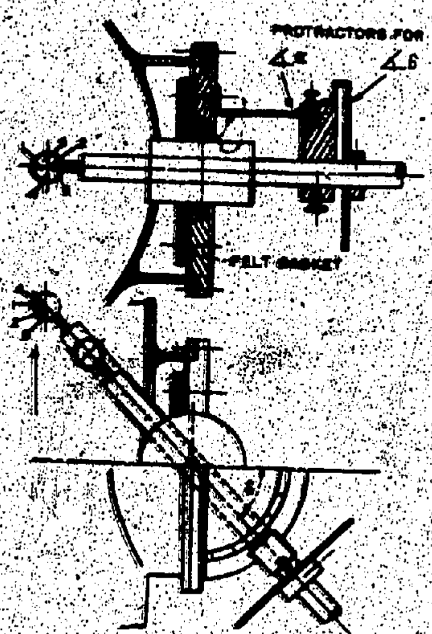

FIG. 4 MOUNTING FOR CALIBRATION OF THE PITOT SPHERE

FIG. 5 CALIBRATION CURVES FOR A PITOT SPHERE

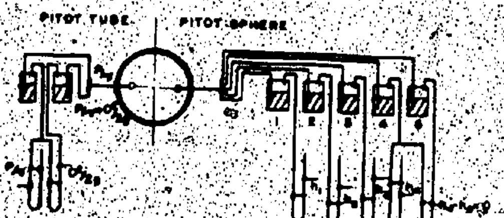

FIG. 6 MANOMETER ARRANGEMENT FOR CALIBRATION

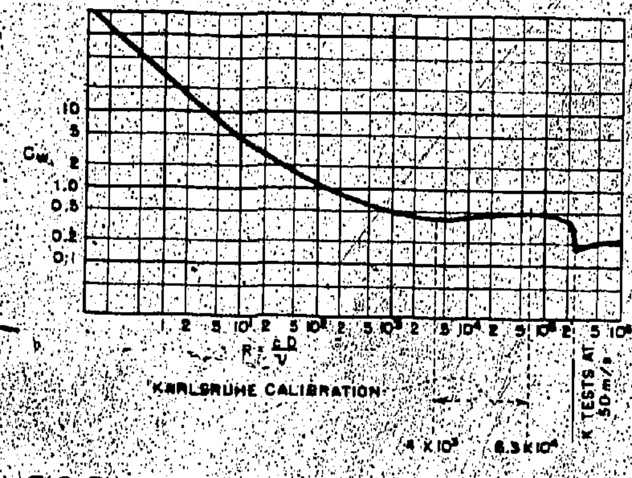

FIG. 7 – RESISTANCE COEFFICIENT OF THE SPHERE

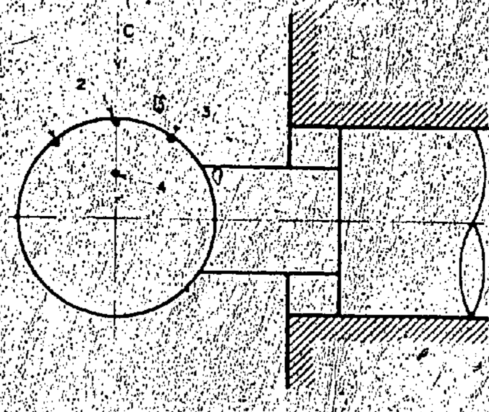

FIG. 8

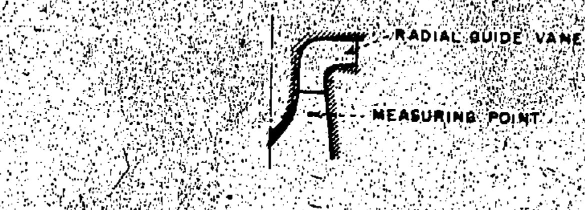

FIG. 9 – VELOCITY AND PRESSURE DISTRIBUTION IN MERIDIONAL FLOW

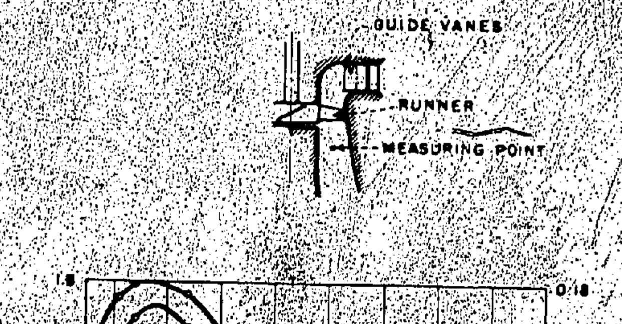

FIG. 10 – VELOCITY AND PRESSURE DISTRIBUTION BELOW A TURBINE RUNNER