SUBMERGED FLOW AT AN OPEN CHANNEL GATE

by

Glen O. Ames

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Civil and Environmental Engineering

Approved:

Roland W. Jeppson
Major Professor

William J. Rahmeyer
Committee Member

Joseph A. Caliendo
Committee Member

James P. Shaver
Dean of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

1999

UMI Number: 1397144

Bell & Howell Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

ABSTRACT

Submerged Flow at an Open Channel Gate

by

Glen O. Ames, Master of Science

Utah State University, 1999

Major Professor: Dr. Roland W. Jeppson
Department: Civil and Environmental Engineering

Two types of flow may exist at an open channel gate: free and submerged. In free flow, the gate creates a subcritical upstream depth, a supercritical jet depth, and, if the channel continues with a mild slope, a tailwater depth that is preceded by a hydraulic jump. In submerged flow, the momentum function associated with the tailwater depth becomes greater than the momentum function associated with the jet, which causes the jet to become submerged under a swirling roller. Submerged flow is controlled by the upstream depth, the gate opening, and the downstream submergence depth, whereas free flow is only controlled by the upstream depth and the gate opening.

The focus of this study is limited to submerged flow. Free flow is examined, but only as a precursor to submerged flow. For example, the special energy and momentum equations are developed for free flow, then modified for submerged flow.

The special specific energy equation is used to predict submerged flow. The development of the equation consists of equating the upstream specific energy to the downstream "special" specific energy. The upstream specific energy is the sum of the upstream depth and the upstream velocity head. The downstream special specific energy is the sum of the downstream submergence depth (consisting of the jet depth and the overlying roller) and the velocity head for the jet. Comparison of computed and measured flows shows agreement to within 1.6%, without a correction coefficient.

The special momentum equation is used to model the flow downstream of the gate, where the jet dissipates and the water surface rises from the submergence depth to the tailwater depth. This development of the special momentum equation is accomplished by equating the special momentum function (submergence depth in the hydrostatic term and jet depth in the dynamic term) to the momentum function associated with the tailwater depth. A coefficient of momentum multiplies the tailwater depth, which accounts for two- and three-dimensional flow effects. Using the value of 1.0 for the coefficient gives agreement to within 2.1%, between computed and measured depths. With a coefficient equal to 1.015, the difference between computed and measured depths is lowered to 1.2%.

(80 pages)

ACKNOWLEDGMENTS

A huge thanks goes out to Dr. Jeppson, who teaches comprehensively, helps freely, and never gives up. This thesis is rooted in his knowledge of open channel flow.

Gratitude is extended to Dr. Rahmeyer for sparking my interest in fluid mechanics from the very start.

A special thanks to Dr. Caliendo for his clear way of teaching and sense of humor. It kept me from getting discouraged throughout my academic journey.

Thanks to all my family and friends for their support and encouragement.

Thanks to my beautiful wife, Nicole, for patiently supporting me through 7 years of study, while living in the low-rent district known as Aggie Village. Thanks also to Zachary and Abigail for being patient with their dad.

Glen O. Ames

CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Water is critical to the well-being of mankind because of its large scope of use. Food production, animal habitat, municipal and industrial use, recreation and lemonade are only a few of its benefits. Water is as important as the sun itself for the sustenance of life on earth. As the human population grows, demand for water grows with it, and local demand often exceeds local supply. Therefore, it is important to measure water accurately for fair allocation.

Man-made water distribution systems utilize pipes and open channels, and the flow may be measured in either case. Open channel measurement has been performed in a variety of ways, as shown in Table 1. Methods range from floating soda cans to ultrasonic metering, and there is a large range of costs and accuracies. Some of the methods employ portable devices and others use permanent structures. Some may be left alone for a period of time, as they take continuous readings, while others require that an operator be present to take the readings manually.

Table 1. Measurements methods for open channel flow

Observation	Pitot Tube	Flume
Floating Object	Propeller Meter	Weir
Dye Injection	Ultrasonic Meter	Salt Dilution
Radioisotope	Deflection Meter	Salt Velocity

Gates are the structures which *control* flow and depth in open channel distribution systems, while the flow is typically *measured* separately with weirs, flumes, etc. Cost may be reduced, however, if the gate is used for both control and measurement. The trade-off may be lower accuracy in flow data. To use a gate for flow measurement, it is typically calibrated using hydraulic theory coupled with field data analysis specific to each gate. This study seeks standard equations that accurately model submerged flow and key depths, without the need for calibration.

Often in water delivery, a series of gates is used to control flows and keep depths large enough to divert water into the turnout structures between these gates. When this occurs, the downstream flow will probably submerge the gate bottom, causing "submerged flow". In this case, a jet of water flowing from under the gate is submerged by a depth of water which has a near zero net movement, because it is swirling. Another condition, which may exist immediately downstream from the gate, is called "free flow." This is where the jet of water has a free surface. Therefore, there are two types of flow that may occur at an open channel gate: free and submerged. If the gate bottom is above the water surface, the flow is unaffected. These conditions are shown in Figure 1.

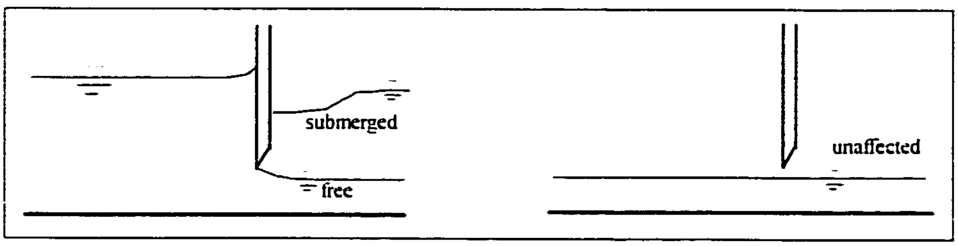

Figure 1. Submerged flow, free flow, and unaffected flow.

Table 2 and Figure 2 show the depths involved in both flow types. The depth upstream of the gate is Yu. The gate opening, Yg, is related to the jet, Yj, through the gate contraction coefficient, or Yj=CcYg. For submerged flow, the submergence depth, Yd, overrides the jet. For free flow, the downstream depth increases along an 'M3' gradually varied flow profile to the depth, Y2, immediately upstream from a hydraulic jump. The downstream tailwater depth, Yt, is controlled by downstream conditions.

This study focuses on submerged flow, although it is helpful to understand free flow, because it has some characteristics similar to submerged flow. Both flow conditions have an upstream depth, a jet depth, and a tailwater depth. Free flow is controlled by Yu and Yg, whereas submerged flow is also affected by Yd. The visual difference between the two flow types is that the jet is submerged for submerged flow.

Table 2. Free flow vs. submerged flow

Free Flow	Submerged Flow
Depths: Yu, Yg, Yj, Y2, Yt	Depths: Yu, Yg, Yj, Yd, Yt
Yj has a free surface	Yj is submerged under Yd
Flow controlled by Yu and Yj	Flow controlled by Yu, Yj and Yd

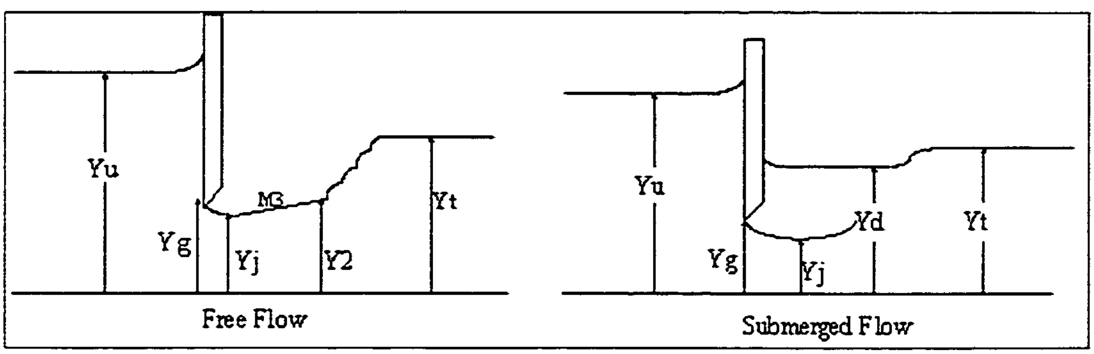

Figure 2. Free flow vs. submerged flow.

The flow will change from free to submerged when the downstream momentum function is larger than the special momentum function, which is computed from the jet depth and the submergence depth (Jeppson, 1994).

Flow past the gate is accelerated rapidly, and the curvature of the streamlines is too large to ignore; therefore, the classification of both flows is "rapidly varied flow" (RVF). The methods of one-dimensional hydraulics have usually been complemented with experimentally determined coefficients to correct for two- and three-dimensional effects on RVF flow patterns. Better theory can be developed using two-dimensional, or three-dimensional formulations (Jeppson, 1994).

The purpose of this thesis is to develop a set of one-dimensional equations that are accurate enough to calculate submerged flow and depth to within 5% accuracy. This will be accomplished by examining the control volume of fluid in the vicinity of the gate, and developing the necessary equations using one-dimensional conservation of energy, momentum, and mass.

CHAPTER II

OPEN CHANNEL GATES

Open channel gates may be classified as: overflow, underflow, or both over and underflow. The most common underflow gates are: vertical (sluice), radial (Tainter), and drum.

Both overflow and underflow gates control flow and depth. However, the upstream depth is more closely related to the height of an overflow gate while underflow gates are best for flow control. Both types serve a variety of purposes. They appear at channel entrances from reservoirs, at crests of overflow spillways, and in the channels themselves, as in-line or turnout structures.

The choice of gate depends on many factors, and each gate has its own advantages. The vertical gate, for example, requires a roller and track assembly which transmits thrust to the sidewalls. The Tainter gate requires structural strength where the thrust is concentrated at the hinge (Henderson, 1966).

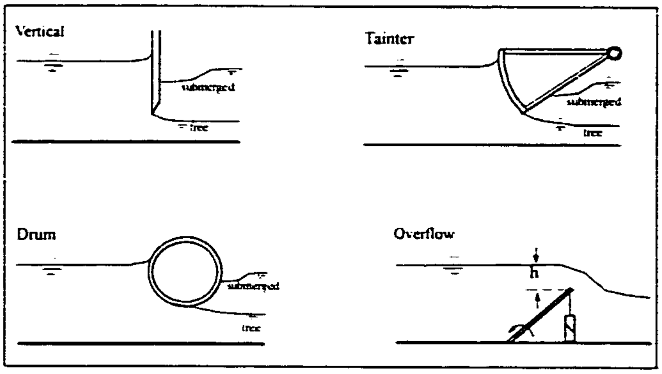

Figure 3. Open channel gate types.

Variations of these gates exist, and have been developed for such purposes as to automatically adjust their positions to keep constant upstream depths or constant downstream depths, by the use of floats attached to the gate. Three of these gates are the AMIL, AVIS, and AVIO gates, which were developed by French engineers from the NEYRTEC company. The AMIL gate is designed to maintain a constant upstream depth, and uses a float on the upstream face of the gate leaf to adjust the gate's height. The AVIS and AVIO gates are designed to maintain a constant downstream depth by means of a float attached downstream from the gate's axis of rotation (Jeppson, 1994). These gates are shown in Figure 4.

The treatments in the following chapters deal specifically with flow past vertical gates because they are most common. However, when the conditions are similar to those described for other gates, then these results can be used for other gate types. These basic conditions are that the fluid exits the gate as a jet that flows under the downstream flow with an overriding roller, and that submergence depth immediately downstream from the gate is less than the tailwater depth further downstream. This difference is caused by the dissipation of the jet (Jeppson, 1994).

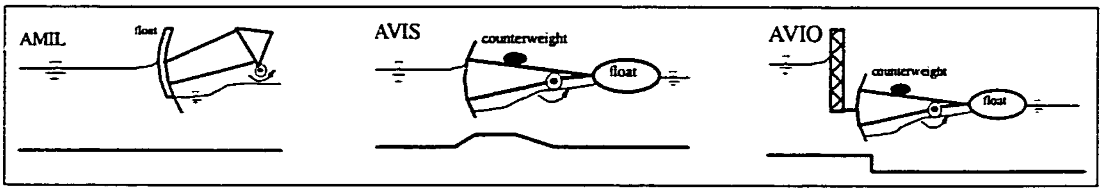

Figure 4. Depth-controlling french gates.

CHAPTER III

DEVELOPMENT OF THE EQUATIONS

Background

The mathematical equations which model fluid flow are developed through the use of principles of fluid mechanics. The three fundamental principles are: conservation of mass, conservation of energy, and conservation of momentum. Another important principle is Newton's second law of motion, as applied in fluid mechanics. This chapter shows the development of the special specific energy and special momentum equations through the use of these principles. In addition, this chapter shows the equation which represents the combination of these equations, and the equation which describes the contraction of the jet.

Energy

The energy contained in fluid flow at an open channel gate will now be examined. First, it is assumed that the flow is incompressible, i.e., the fluid density does not change. Second, the flow is assumed to be steady-state (unchanging with time). Third, the energy loss at the gate is assumed to be negligible.

The analysis begins with the Euler equation, $\frac{\partial}{\partial s}(P+\gamma Z)+\rho a_s=0$, where P=pressure, γ=specific weight, Z=elevation, ρ=density, a_s=acceleration in the 's' direction. Assuming steady flow, the equation becomes: $\frac{\partial}{\partial s}(P+\gamma Z)+\rho v\frac{\partial v}{\partial s}=0$, where v=velocity. If this equation is integrated with respect to direction, the Bernoulli equation is obtained: $P+\gamma Z+\frac{1}{2}\rho v^2=C$, or $\frac{P}{\gamma}+Z+\frac{v^2}{2g}=C$, where C=constant.

All units in the Bernoulli equation are in terms of length, which is equivalent to energy per unit weight. This dimensional analysis is shown below.

$$\frac{\text{Kinetic Energy}}{\text{Weight}} = \frac{\frac{1}{2}\text{mass}\times\text{Velocity}^2}{\text{mass}\times\text{gravity}} = \frac{V^2}{2g} = \frac{\frac{L^2}{t^2}}{\frac{L}{t^2}} = \text{length}$$

$$\frac{\text{Energy}}{\text{Weight}} = \frac{\text{Pressure}\times\text{Area}\times\text{Distance}}{\text{Weight}} = \frac{P}{\gamma} = \frac{\frac{F}{L^2}}{\frac{F}{L^3}} = \text{length}$$

'Z' is already in terms of length because it represents potential energy due to elevation.

Assume that C = H (total head), or $\frac{P}{\gamma} + z + \frac{V^2}{2g} = H$.

Applying this equation across the gate, assuming no energy loss gives:

$$\frac{P_u}{\gamma} + z_u + \frac{V_u^2}{2g} = \frac{P_j}{\gamma} + z_j + \frac{V_j^2}{2g}.$$

Now, let fluid depth = piezometric head, $Y = \frac{P}{\gamma} + Z$. This results in the following "specific" energy equation:

$$Y_u + \frac{V_u^2}{2g} = Y_j + \frac{V_j^2}{2g}.$$

Since $V = \frac{Q}{A}$, and $Q_u = Q_j$ (conservation of mass, or continuity), then:

$$Y_u + \frac{Q^2}{2gA_u^2} = Y_j + \frac{Q^2}{2gA_j^2}.$$

For a trapezoidal cross section (see Figure 5), where $A = bY + mY^2$, the specific energy equation becomes:

$$Y_u + \frac{Q^2}{2g(bY+mY^2)_u^2} = Y_j + \frac{Q^2}{2g(b_g Y_j)^2},$$ where bg is the gate width.

For a rectangular cross section (see Figure 6), where $A = bY$ and $q = Q/b$ (flow per unit width), the equation becomes:

$$Y_u + \frac{q^2}{2gY_u^2} = Y_j + \frac{q^2}{2gY_j^2}.$$

Rearranging the equation to isolate flow yields Equation 3.1.

$$q = \frac{Y_u Y_j \sqrt{2g(Y_u - Y_j)}}{\sqrt{Y_u^2 - Y_j{}^2}} \tag{3.1}$$

Equation 3.1 applies to free flow at an open channel gate. It represents conservation of energy between positions 1 and 2. It is assumed that the energy loss across the gate is negligible. Figure 7 shows the free flow diagram. Yu is the upstream depth. Yg is the height of the gate opening, Yj is the jet depth measured at the lowest point. M3 is the gradually varied profile (GVF) curve which leads to the alternate depth, Y2. and the depth Yt follows the hydraulic jump.

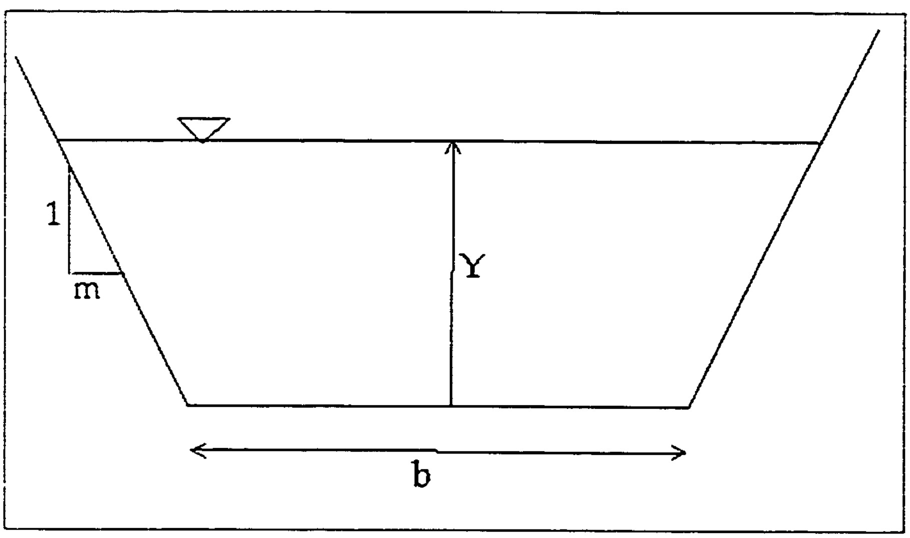

Figure 5. Trapezoidal cross section.

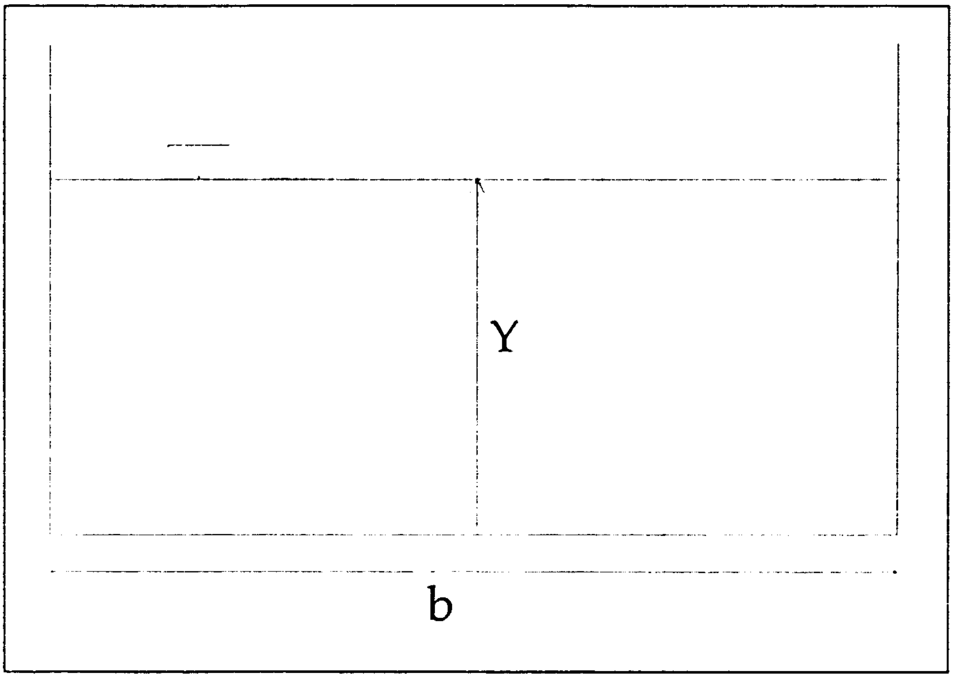

Figure 6. Rectangular cross section.

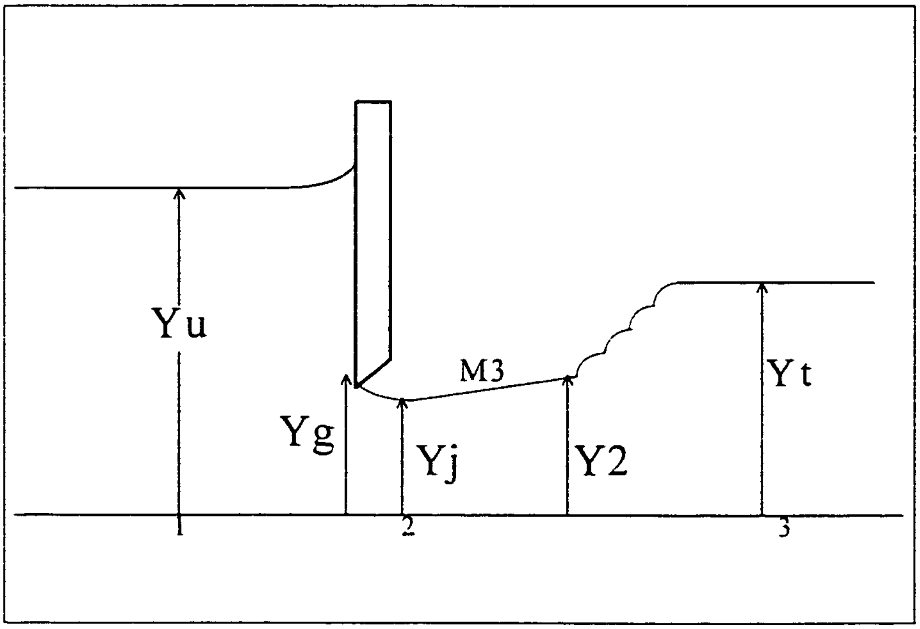

Figure 7. Free flow.

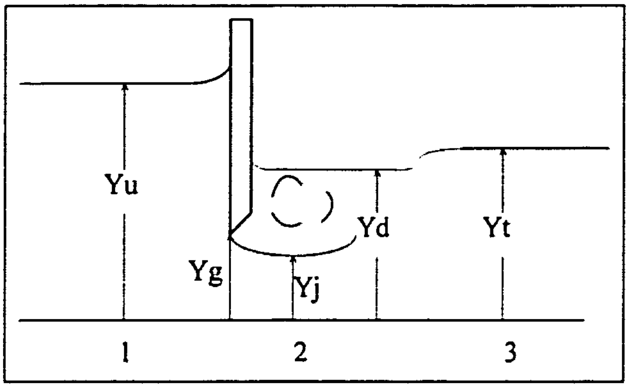

Figure 8. Submerged flow.

For submerged conditions (see Figure 8), Yd represents the submergence depth against the downstream side of the gate and Yj represents the jet depth. To modify the specific energy equation for submerged conditions, Yd remains in the static term, but the jet depth is placed in the dynamic term. Assuming there is negligible energy loss for the fluid in the jet, the following "special" specific energy equation is produced:

$$\underbrace{Y_u}_{\text{Static}} + \underbrace{\frac{q^2}{2gY_u^2}}_{\text{Dynamic}} = \underbrace{Y_d}_{\text{Static}} + \underbrace{\frac{q^2}{2gY_j^2}}_{\text{Dynamic}}.$$

Rearranging to isolate flow on one side of the equation yields:

$$q = \frac{Y_u Y_j \sqrt{2g(Y_u - Y_d)}}{\sqrt{Y_u^2 - Y_j^2}} \tag{3.2}$$

This equation applies to submerged flow at an open channel gate. It represents conservation of energy between positions 1 and 2. Yu and Yd may be measured with staff gauges installed in the channel bank, and Yj is equal to the gate opening multiplied by Cc. Thus, the most practical application of Equation 3.2 is to calculate flow, q. Verification of this equation is found in Chapter V.

Momentum

Now, we will examine the flow between positions 2 and 3, where the depth Yt is introduced. If the flow is free, there will be a hydraulic jump downstream, but in the case of submerged flow, the jet from under the gate causes a modest increase in depth to occur. The downstream tailwater depth, Yt, is larger than the depth, Yd, which overrides the jet.

In the field, it is more practical to measure Yt than Yd. Therefore, there are now two unknowns: q and Yd. Since there are two unknowns, there must be two independent equations to solve the problem. However, another energy equation may not be used between positions 2 and 3 because there is an unknown energy loss at this portion of the flow. The use of momentum theory is valuable when there is energy loss, forces control the direction or conditions associated with fluid motions, or when it is not possible to explain what is happening to the fluid on a microscopic level, but a large control volume of fluid is possible (Jeppson, 1994).

Since this is a case where there is energy loss, a momentum equation will be used to describe the flow between positions 2 and 3. Let m represent the momentum function per unit width, M/b. The momentum function is obtained by examining a hydraulic jump between positions 2 and 3. Move a short distance downstream and upstream of the jump, remove the fluid outside the boundary, and replace it with equivalent forces, as shown in Figure 9.

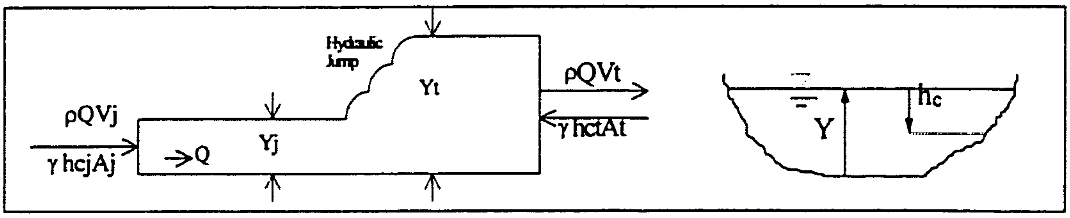

Figure 9. Momentum principle at a control volume.

The hydrostatic forces equal γhcA, where hc is the depth to the centroid of the cross sectional area. The momentum fluxes equal ρQV, where ρ is the density of the fluid. The difference in hydrostatic forces (upstream minus downstream) must equal the difference in the momentum fluxes (downstream minus upstream), which gives:

$$\gamma h_{cj}A_j - \gamma h_{ct}A_t = \rho Q(V_t - V_j).$$

Dividing by γ and substituting V=Q/A gives:

$$h_{cj}A_j - h_{ct}A_t = \frac{Q^2}{g}\left(\frac{1}{A_t} - \frac{1}{A_j}\right).$$

For a rectangular cross section, the equation becomes:

$$\frac{Y_j^2}{2} - \frac{Y_t^2}{2} = \frac{Q^2}{g}\left(\frac{1}{b_tY_t} - \frac{1}{b_jY_j}\right).$$

Assuming the bed width does not change, the equation becomes:

$$\underbrace{\frac{Y_j^2}{2} - \frac{Y_t^2}{2}}_{\text{Static}} = \underbrace{\frac{q^2}{g}\left(\frac{1}{Y_t} - \frac{1}{Y_j}\right)}_{\text{Dynamic}}.$$

Assuming there is no net motion in the fluid above Yj, the equation is modified for submerged flow by replacing Yj with Yd in the static term. This results in the following "special" momentum equation:

$$\frac{Y_d^2}{2} - \frac{Y_t^2}{2} = \frac{q^2}{g}\left(\frac{1}{Y_t} - \frac{1}{Y_j}\right).$$

Equation 3.3 applies to the section between positions 2 and 3, in a rectangular channel with constant bed width. Rearranging the equation to solve for flow yields a simplified form.

$$q = \sqrt{\frac{gY_jY_t(Y_t^2 - Y_d^2)}{2(Y_t - Y_j)}} \tag{3.3}$$

Transition from Free Flow to Submerged Flow

A transition from free flow to submerged flow occurs when the momentum function associated with the tailwater depth, $\frac{Y_t^2}{2} + \frac{q^2}{gY_t}$, becomes equal to or greater than the momentum function of the jet, $\frac{Y_j^2}{2} + \frac{q^2}{gY_j}$ (Jeppson, 1994). Therefore, for a rectangular channel, submerged flow will exist at the gate if:

$$\frac{Y_t^2}{2} + \frac{q^2}{gY_t} \geq \frac{Y_j^2}{2} + \frac{q^2}{gY_j} .$$

The momentum function for a trapezoidal channel is defined as the following:

$$M = (\frac{bY^2}{2} + \frac{mY^3}{3}) + \frac{Q^2}{g}(\frac{1}{bY+mY^2}).$$

In this case of a trapezoidal channel, the momentum function for the jet contains both trapezoidal and rectangular geometry because the upstream section is trapezoidal and the jet assumes the rectangular shape of the gate opening, as shown in the equation below. The flow will become submerged if the left side of the equation becomes equal to or greater than the right side of the equation.

$(\frac{b_tY_t^2}{2} + \frac{m_tY_t^3}{3}) + \frac{Q^2}{g}(\frac{1}{b_tY_t+m_tY_t^2}) \geq \frac{b_gY_j^2}{2} + \frac{Q^2}{g}(\frac{1}{b_gY_j})$, where bg is the gate width.

Boiten (1992, p. 8) wrote, "The limit at which the hydraulic jump will submerge the free jet is called the modular limit." He uses the following equation to describe the limit mathematically:

$$\frac{Y_t}{Y_g} = \frac{C_c}{2}\left[\sqrt{1 + 16\left(\frac{Y_u}{C_c Y_g} - 1\right)} - 1\right].$$

This equation is useful since only depths are involved, which allows for any cross section. Figure 10 shows the graphical representation of the equation. The value of Cc=0.611 is used for a sharp-edged gate, where the edge diameter is zero (d=0). The value of Cc= 0.990 is used for a rounded edge, where d > 4.7*Yg .

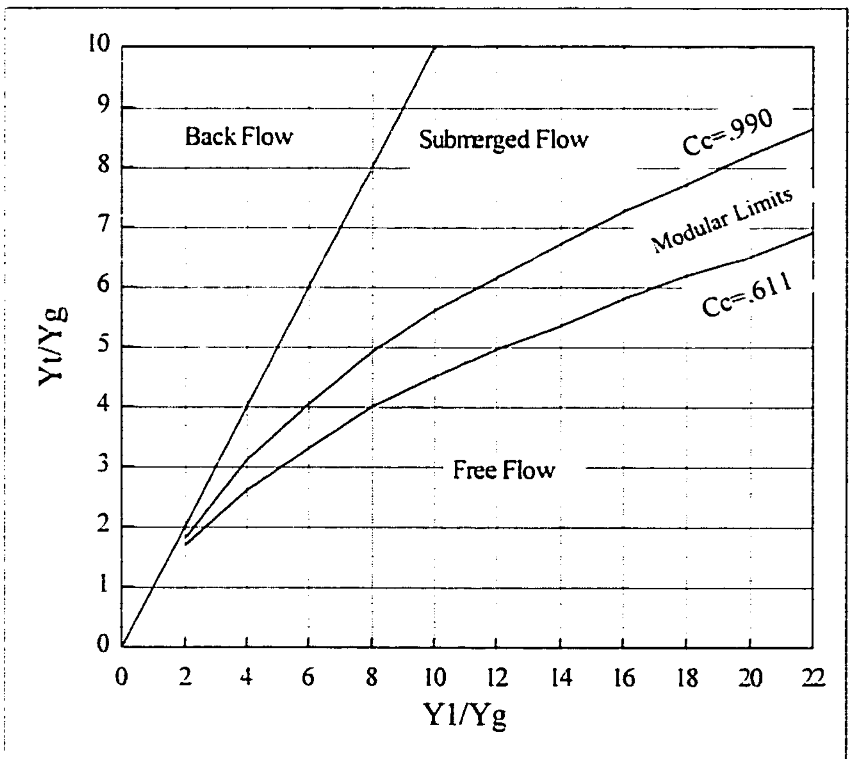

Figure 10. Modular limit.

Combination

Since the special specific energy and special momentum equations have the flow term isolated on the left side, they may be combined into a quadratic equation, which eliminates flow and gravity in the equation.

$$\frac{Y_t(Y_u^2-Y_j^{\,2})(Y_t^2-Y_d^{\,2})}{4Y_u^2Y_j(Y_t-Y_j)}+Y_d-Y_u=0 \tag{3.4}$$

The variables in Equation 3.4 are the four depths associated with submerged flow. The equation incorporates the conservation of energy, momentum, and mass. It may be used to find any of the four depths, although it is more practical to use it for the calculation of Yu. The depths Yd and Yt may be measured with staff gauges, and Yj is found by multiplying the gate opening by the coefficient of contraction. Equation 3.4 will be tested for validity in Chapter V.

Contraction

Assuming no energy loss in the fluid at the gate, specific energy will not change across the gate. Henderson (1966) explains that for a given specific energy and flow, there are two possible depths in the channel. These are called "alternate" depths, and for this case they would be Yu and Yj. Since Yj<Yu, the velocity of the jet must be greater than the upstream velocity for the specific energies to be equal.

As the water flows from the sharp-edged gate opening, the jet contracts to a minimum section, known as the vena contracta. This occurs at a distance roughly equal to Yg from the gate (Rajaratnam and Subramanya, 1967). When the cross sectional area decreases, the streamlines in the moving fluid get closer together in order to fit through the gate opening (see Figure 11). The lower streamlines are somewhat parallel with the direction of flow, while the upper streamlines are at a steeper angle. The angle of these upper streamlines causes the fluid to contract as it emerges from the opening.

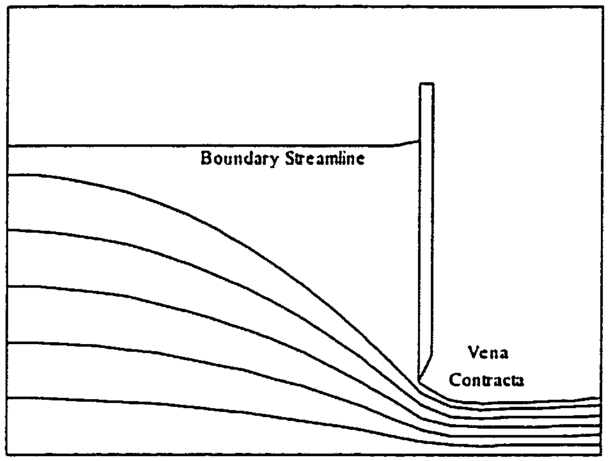

Figure 11. Jet contraction from streamline curvature.

The coefficient of contraction will be defined as a fraction of the gate setting. For example, if the gate setting is 10 inches, and the coefficient of contraction is 0.6, then the jet depth is equal to 6 inches. It is convenient to substitute the expression CcYg in lieu of Yj throughout the flow equations. The value of Cc depends on the orifice through which the fluid exits. Potential flow theory has produced a value of 0.583 for Cc, for a two-dimensional slot, and Henderson (1966) shows a range of Cc from 0.611 to 0.598 for given Yg/Yu ratios.

Cc should not be completely constant for a sharp-edged channel gate. Woycicki (1934) experimentally determined that Cc equals a constant plus 0.04 multiplied by the ratio of gate setting to upstream depth. It makes sense that Cc varies with the ratio Yg/Yu because the greater the difference between the two depths, the more contraction there should be. By using 0.583 as the lower limit, and adding the variation due to the head difference across the gate, the following equation is produced.

$$C_c = 0.583 + 0.04\frac{Y_g}{Y_u} \tag{3.5}$$

To see how this equation behaves, imagine an infinite depth, Yu. The value of Cc will be 0.583, as the ratio approaches zero. Because the upper streamlines are coming down from a steeper angle, the fluid contraction will be strong. On the other hand, if the upstream depth decreases to the point that it almost equals the gate opening, the coefficient approaches a value of 0.623. For radial gates, the coefficient of contraction would also vary with its radius, r (Jeppson, 1994).

General Equations

Equations 3.2, 3.3, and 3.4 may only be used for open channels which have a rectangular cross section, but most channels actually have a trapezoidal cross section because there is greater stability on the side slopes. An equation for a trapezoidal channel may be used for a rectangular channel if the side slope, m, is equal to zero. Therefore, the general equations will be presented in trapezoidal form. Also, it is convenient to set the equations equal to zero, when solving for variables.

Special specific energy equation between positions 1 and 2:

$$Y_u - Y_d + \frac{Q^2}{2g}\left(\frac{1}{(b_1 Y_u + m_1 Y_u^2)^2} - \frac{1}{(b_g Y_j)^2}\right) = 0$$

Special momentum equation between positions 2 and 3:

$$\tfrac{1}{2}b_2(Y_d^2 - Y_t^{\ 2}) + \tfrac{1}{3}m_2(Y_d^3 - Y_t^3) + \frac{Q^2}{g}\left(\frac{1}{(b_2 Y_t + m_2 Y_t^2)} - \frac{1}{(b_g Y_j)}\right) = 0$$

Combination equation between positions 1 and 3:

$$\frac{Y_t(Y_u^2 - Y_j^{\ 2})(Y_t^{\ 2} - Y_d^{\ 2})}{4Y_u^2 Y_j (Y_t - Y_j)} + Y_d - Y_u = 0$$

Coefficient of contraction:

$$C_c = 0.583 + 0.04\frac{Y_g}{Y_u}$$

Jet depth:

$$Yj = Yg(0.583 + 0.04\frac{Y_g}{Y_u}) \qquad (3.6)$$

CHAPTER IV

LITERATURE REVIEW

There are many authors who have studied submerged flow at an open channel gate. Some of these include Henry (1950), Dirkzwager (1958), Henderson (1966), Garbrecht (1980), Kolkman (1989), Boiten (1992), and Jeppson (1994). The flow equations are as varied as the authors themselves, and this chapter will briefly discuss some of them.

Boiten claims that the most common equation for submerged flow is: $Q = C2 Y_g b \sqrt{2gY_u}$, where C2 is found from Figure 12 by Kolkman (1989). Kolkman computed values of C2 that agree well with measurements by Henry. Values for C2 may range between 0 and 1.0, depending on the degree of submergence. This equation differs from Equation 3.2 in that the head difference, Yu-Yd, is neglected in the radical term. The explanation why Kolkman's equation gives reasonable estimates for Q is that C2 accounts for the head difference, as seen in Figure 12.

Another equation is: $Q = C1\, Y_g b \sqrt{2g(Y_u - Y_t)}$, where C1 is a function of Yu/Yg, Yt, and the bottom edge shape. Values for C1 may range as follows: 0.8 < C1 < 1.5.

Another equation, presented by Garbrecht, is based on experiments with a sharp-edged gate, and is written as follows: $Q = 0.635 Y_g b \sqrt{2g(Y_u - Y_d)}$. This equation, which uses a coefficient of 0.635, may only be used for sharp-edged gates.

The flow equation by Dirkzwager is similar to the equation by Kolkman, and is written as follows: $Q = CsCdY_g b \sqrt{2gY_u}$. Cd is the flow coefficient for free flow, and Cs is the coefficient of submergence, which depends on Yu/Yg, Yt/Yg and the contraction coefficient, which equals 0.611.

is the coefficient of submergence, which depends on Yu/Yg, Yt/Yg and the contraction coefficient, which equals 0.611.

Boiten recommends the equation by Kolkman for three reasons. The equation agrees well with theoretical expressions and experimental data, all gate bottom shapes are accommodated, and the measurement of downstream head, Yt, is feasible. (Boiten, 1992). The approximate error using this equation to calculate flow is 10%, as opposed to the percent difference of Equation 3.2, which is around 1%.

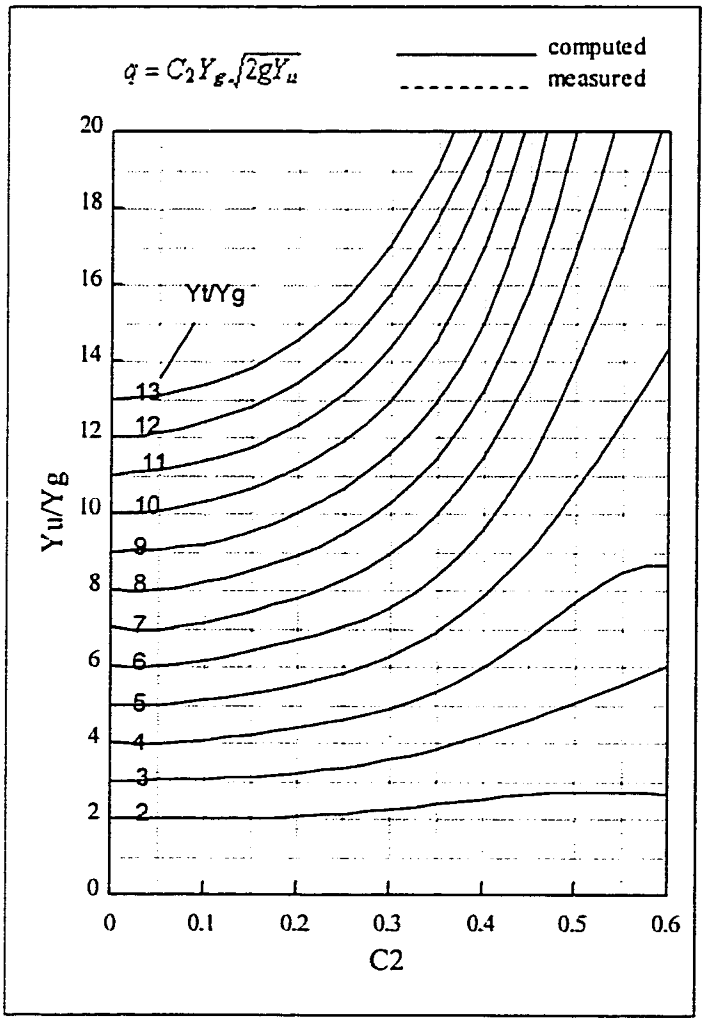

Figure 12. Flow coefficient for submerged 2-D flow through a sharp-edged gate.

CHAPTER V

VERIFICATION OF THE EQUATIONS

Equations 3.2, 3.3, 3.4, and 3.5 will be verified by calculating a variable, while using measured values for the other variables, then checking against the measured value. The writer's goal is to calculate results that are within 5% of past reported measured values, before resorting to calibration.

Table 3 shows laboratory data taken from the report by Rajaratnam and Subramanta (1967). The experiments were conducted in a recirculating flume, 18 inches wide, 36 inches deep, and 16 feet long. The flume had a smooth aluminum bed and vertical side walls of Plexiglas. The sluice gate was an aluminum plate, 0.25 in. thick, with a sharp lower edge. The flow entered a head tank, and the tailwater was controlled by means of another sluice gate located at the downstream end of the flume. The depths Yu and Yd were measured on a manometer board, and Yt was measured with a precision point gage. The flow was measured using a commercial orifice-meter located in the supply line (Rajaratnam and Subramanya, 1967).

For each gate setting, Yg, there are multiple experiments, which include the upstream depth, Yu, the downstream depth, Yd, the tailwater depth, Yt, and flow, q. Notice that for a given flow, there may be multiple sets of depths. This is predicted by Equation 3.2, which states that submerged flow is controlled by the depth difference. The experiments in italics show the same difference (Yu-Yd) of 0.367 ft and the same flowrate of 0.735 cfs/ft.

Table 3. Measured laboratory data (Rajaratnam and Subramanya,1967)

Yg ft	Yu ft	Yd ft	Yt ft	q cfs/ft
0.25	1.217	0.867	0.930	0.735
0.25	*1.550*	*1.183*	1.214	*0.735*
0.25	1.808	1.433	1.473	0.735
0.25	2.025	1.650	1.676	0.730
0.25	2.475	2.125	2.140	0.723
0.25	*1.200*	*0.833*	0.915	*0.735*
0.25	2.092	1.450	1.535	0.950
0.25	2.608	1.975	2.030	0.937
0.25	1.992	1.354	none	0.945
0.25	1.704	1.071	1.183	0.955
0.25	1.500	0.858	0.996	0.960
0.25	1.308	0.675	0.845	0.960
0.25	1.158	0.529	0.742	0.960
0.25	1.042	0.400	0.675	0.960
0.50	0.650	0.525	0.590	0.965
0.50	0.883	0.742	0.817	0.965
0.50	1.100	0.942	1.008	0.965
0.50	1.425	1.267	1.308	0.965
0.50	1.158	1.000	1.058	0.962
0.50	1.633	1.475	1.490	0.950
0.50	0.725	0.596	0.663	0.965
0.33	1.017	0.600	0.785	1.035
0.33	0.800	0.400	0.620	1.035
0.33	1.142	0.725	0.878	1.035
0.33	1.592	1.175	1.271	1.025
0.33	1.883	1.458	1.533	1.025
0.17	2.133	1.108	1.248	0.811
0.17	2.608	1.633	1.720	0.805
0.17	1.608	0.558	0.815	0.823
0.17	1.225	0.417	0.685	0.710
0.17	0.583	0.175	0.404	0.523
0.17	0.725	0.242	0.477	0.518
0.08	1.100	0.360	0.496	0.341
0.08	1.508	0.767	0.841	0.341
0.08	1.725	0.850	0.910	0.374
0.08	2.467	1.617	1.642	0.374
0.08	3.508	1.017	1.190	0.640

$$q = \frac{Y_u Y_j \sqrt{2g(Y_u - Y_d)}}{\sqrt{Y_u^2 - Y_j^2}} \tag{3.2}$$

$$C_c = 0.583 + 0.04\frac{Y_g}{Y_u} \tag{3.5}$$

Equation 3.2 represents conservation of energy between positions 1 and 2, and is used to calculate q as a function of Yu, Yj, and Yd. It will be verified by comparing calculated flows with measured flows. Equation 3.5 will be verified at the same time by replacing Yj with CcYg.

Table 4 shows the results of the verification process. The first three columns contain original lab data from Table 3. The fourth column, Yu-Yd, is used for Equation 3.2. The fifth column shows Cc, as calculated by Equation 3.5. The sixth column calculates jet depth by using Yj=CcYg. The seventh column contains the measured laboratory flows from Table 3. The eighth column shows the calculated flows from Equation 3.2. Finally, the last column shows the percent difference between calculated and measured flows.

The calculated flows agree well with the measured flows, as seen in the last column of Table 4, except for an outlier where the difference was 9.5%. Where q calc is less than q lab, it is shown as a negative value. To find the average percent difference for the entire experiment, all values were changed to absolute values, and then the average of these was found. Table 5 summarizes the information regarding the accuracy of Equations 3.2 and 3.5. The outlier is excluded from the difference calculations.

Table 4. Verification of equations 3.2 and 3.5

Yg ft	Yu ft	Yd ft	Yu-Yd ft	Cc	Yj ft	q lab cfs/ft	q calc cfs/ft	% difference 100(qc-ql)/ql
0.25	1.217	0.867	0.350	0.591	0.148	0.735	0.707	-3.8
0.25	1.550	1.183	0.367	0.590	0.147	0.735	0.720	-2.1
0.25	1.808	1.433	0.375	0.589	0.147	0.735	0.725	-1.3
0.25	2.025	1.650	0.375	0.588	0.147	0.730	0.724	-0.8
0.25	2.475	2.125	0.350	0.587	0.147	0.723	0.698	-3.5
0.25	1.200	0.833	0.367	0.591	0.148	0.735	0.724	-1.5
0.25	2.092	1.450	0.642	0.588	0.147	0.950	0.947	-0.3
0.25	2.608	1.975	0.633	0.587	0.147	0.937	0.938	0.1
0.25	1.992	1.354	0.638	0.588	0.147	0.945	0.945	0.0
0.25	1.704	1.071	0.633	0.589	0.147	0.955	0.943	-1.2
0.25	1.500	0.858	0.642	0.590	0.147	0.960	0.952	-0.8
0.25	1.308	0.675	0.633	0.591	0.148	0.960	0.949	-1.2
0.25	1.158	0.529	0.629	0.592	0.148	0.960	0.949	-1.1
0.25	1.042	0.400	0.642	0.593	0.148	0.960	0.962	0.2
0.50	0.650	0.525	0.125	0.614	0.307	0.965	0.988	2.4
0.50	0.883	0.742	0.141	0.606	0.303	0.965	0.971	0.7
0.50	1.100	0.942	0.158	0.601	0.301	0.965	0.997	3.3
0.50	1.425	1.267	0.158	0.597	0.299	0.965	0.974	0.9
0.50	1.158	1.000	0.158	0.600	0.300	0.962	0.991	3.0
0.50	1.633	1.475	0.158	0.595	0.298	0.950	0.966	1.6
0.50	0.725	0.596	0.129	0.611	0.305	0.965	0.970	0.5
0.33	1.017	0.600	0.417	0.596	0.197	1.035	1.039	0.4
0.33	0.800	0.400	0.400	0.600	0.198	1.035	1.036	0.1
0.33	1.142	0.725	0.417	0.595	0.196	1.035	1.032	-0.3
0.33	1.592	1.175	0.417	0.591	0.195	1.025	1.019	-0.6
0.33	1.883	1.458	0.425	0.590	0.195	1.025	1.024	-0.1
0.17	2.133	1.108	1.025	0.586	0.100	0.811	0.811	-0.1
0.17	2.608	1.633	0.975	0.586	0.100	0.805	0.789	-1.9
0.17	1.608	0.558	1.050	0.587	0.100	0.823	0.822	-0.1
0.17	1.225	0.417	0.808	0.589	0.100	0.710	0.724	2.0
0.17	0.583	0.175	0.408	0.595	0.101	0.523	0.526	0.6
0.17	0.725	0.242	0.483	0.592	0.101	0.518	0.567	9.5*
0.08	1.100	0.360	0.740	0.590	0.050	0.340	0.340	-1.0
0.08	1.510	0.770	0.740	0.590	0.050	0.340	0.340	-1.1
0.08	1.730	0.850	0.880	0.580	0.050	0.370	0.370	-2.1
0.08	2.470	1.620	0.850	0.580	0.050	0.370	0.360	-3.6
0.08	3.510	1.020	2.490	0.580	0.050	0.640	0.620	-3.7

*Outlier

Avg = 1.6%
Max = 3.8%

The percent differences are very small, since other authors claim average differences of between 5% and 15% for submerged flow calculations (Boiten, 1992; Rajaratnam and Subramanya, 1967). The accuracy of Equation 3.2 can be seen graphically by plotting calculated flows against measured flows (see Figure 13).

Table 5. Accuracy summary for equations 3.2 and 3.5

Accuracy Information	Eqns 3.2 & 3.5
Average Difference (%)	1.6
Minimum Difference(%)	0.0
Maximum Difference (%)	3.8

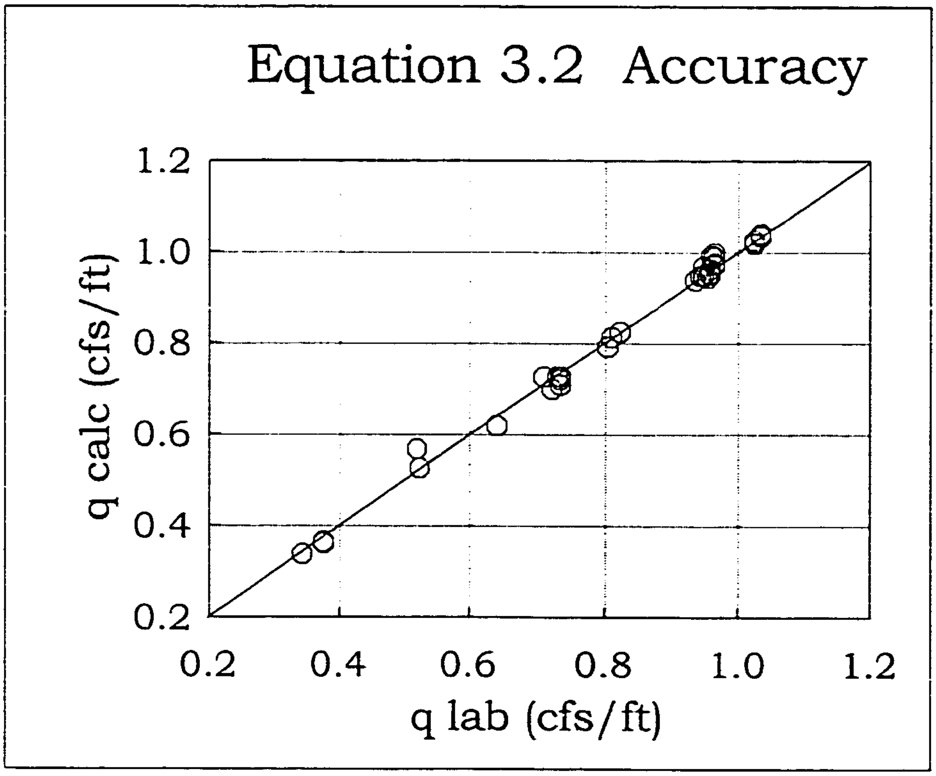

Figure 13. Accuracy of equation 3.2.

The hollow points represent calculated flows, which should fall on the 45-degree line if they agree perfectly with measured data. Points above the line were overestimated and points below the line were underestimated. Note that there is good agreement through the entire range of flows.

$$q = \sqrt{\frac{gY_j C_m Y_t((CmY_t)^2 - Y_d^2)}{2(C_m Y_t - Y_j)}} \qquad (3.3')$$

Equation 3.3' represents conservation of momentum between positions 2 and 3, and is used to calculate tailwater depth as a function of jet depth, downstream depth, and flow. It is different from Equation 3.3 because a coefficient of momentum multiplies tailwater depth. The coefficient is assumed to account for two- and three-dimensional flow effects. Equation 3.3' will be verified by comparing calculated and measured depths of the tailwater.

Table 6 shows the results of the verification process, using a value of 1.0 for the coefficient of momentum. The first three columns contain measured laboratory data. The fourth column shows the coefficient of contraction, as calculated by Equation 3.5. The fifth column calculates jet depth using Yj=CcYg. The sixth column contains measured flows. The seventh column shows the measured tailwater depths. The eighth column shows the calculated values for tailwater depth, using Equation 3.3'. Finally, the last column shows the percent difference between the calculated and measured tailwater depths.

Table 6. Verification of equation 3.3' (Cm=1.0)

Yg ft	Yu ft	Yd ft	Cc ft	Yj ft	q cfs/ft	Yt lab ft	Yt calc ft (Cm=1.0)	% error 100(Ytc-Ytl) / Ytl
0.25	1.217	0.867	0.591	0.148	0.735	0.930	0.957	2.9
0.25	1.550	1.183	0.590	0.147	0.735	1.214	1.246	2.7
0.25	1.808	1.433	0.589	0.147	0.735	1.473	1.481	0.5
0.25	2.025	1.650	0.588	0.147	0.730	1.676	1.686	0.6
0.25	2.475	2.125	0.587	0.147	0.723	2.140	2.141	0.0
0.25	1.200	0.833	0.591	0.148	0.735	0.915	0.927	1.3
0.25	2.092	1.450	0.588	0.147	0.950	1.535	1.542	0.4
0.25	2.608	1.975	0.587	0.147	0.937	2.030	2.030	0.0
0.25	1.704	1.071	0.589	0.147	0.955	1.183	1.201	1.5
0.25	1.500	0.858	0.590	0.147	0.960	0.996	1.019	2.3
0.25	1.308	0.675	0.591	0.148	0.960	0.845	0.869	2.9
0.25	1.158	0.529	0.592	0.148	0.960	0.742	0.758	2.3
0.25	1.042	0.400	0.593	0.148	0.960	0.675	0.670	-0.8
0.50	0.650	0.525	0.614	0.307	0.965	0.590	0.598	1.4
0.50	0.883	0.742	0.606	0.303	0.965	0.817	0.807	-1.2
0.50	1.100	0.942	0.601	0.301	0.965	1.008	0.996	-1.2
0.50	1.425	1.267	0.597	0.299	0.965	1.308	1.305	-0.2
0.50	1.158	1.000	0.600	0.300	0.962	1.058	1.051	-0.7
0.50	1.633	1.475	0.595	0.298	0.950	1.490	1.503	0.9
0.50	0.725	0.596	0.611	0.305	0.965	0.663	0.668	0.8
0.33	1.017	0.600	0.596	0.199	1.035	0.785	0.769	-2.0
0.33	0.800	0.400	0.600	0.200	1.035	0.620	0.612	-1.3
0.33	1.142	0.725	0.595	0.198	1.035	0.878	0.874	-0.5
0.33	1.592	1.175	0.591	0.197	1.025	1.271	1.270	-0.1
0.33	1.883	1.458	0.590	0.197	1.025	1.533	1.531	-0.1
0.17	2.133	1.108	0.586	0.098	0.811	1.248	1.252	0.3
0.17	2.608	1.633	0.586	0.098	0.805	1.720	1.722	0.1
0.17	1.608	0.558	0.587	0.098	0.823	0.815	0.819	0.5
0.17	1.225	0.417	0.588	0.098	0.710	0.685	0.658	-3.9
0.17	0.583	0.175	0.594	0.099	0.523	0.404	0.394	-2.6
0.17	0.725	0.242	0.592	0.099	0.518	0.477	0.428	-10.2*
0.08	1.100	0.360	0.586	0.049	0.341	0.496	0.506	2.0
0.08	1.508	0.767	0.585	0.049	0.341	0.841	0.840	-0.1
0.08	1.725	0.850	0.585	0.049	0.374	0.910	0.930	2.2
0.08	2.467	1.617	0.584	0.049	0.374	1.642	1.645	0.2
0.08	3.508	1.017	0.584	0.049	0.640	1.190	1.221	2.6

*Outlier

Avg = 2.1%
Max = 4.5%

The calculated depths are slightly higher the measured depths for much of the data. Where Yt calc is less than Yt lab, it is shown as negative. To find the average percent difference for the entire experiment, all values were changed to absolute values, then, the average of these was found. Table 7 summarizes the information regarding the accuracy of Equation 3.3' with Cm=1.0. The accuracy of Equation 3.3' can be seen graphically by plotting the measured depths against the calculated depths (see Figure 14).

Table 7. Accuracy summary for equation 3.3' (Cm=1.0)

Accuracy Information	Equation 3.3' (Cm=1.0)
Average Difference (%)	2.1
Minimum Difference (%)	0.2
Maximum Difference (%)	4.5

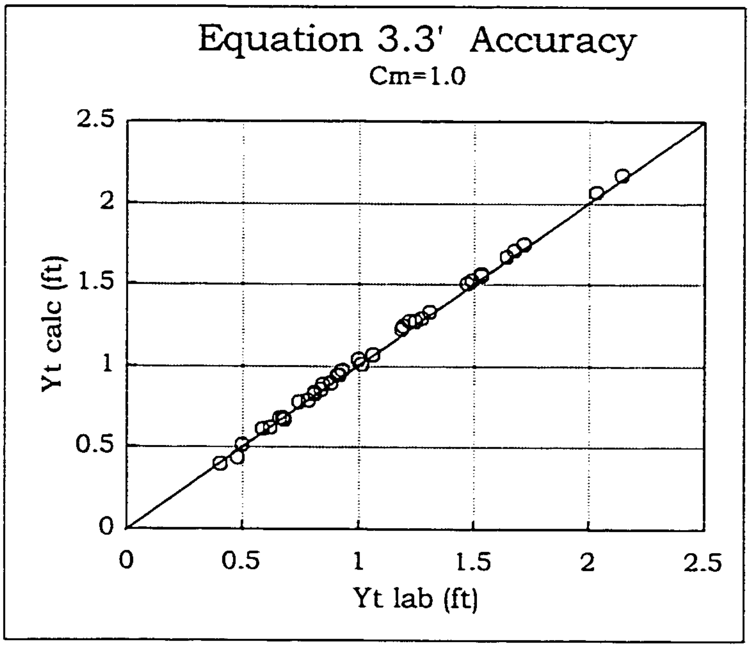

Figure 14. Accuracy of equation 3.3' (Cm=1.0).

The points represent calculated depths, and they should fall on the 45° line if there is perfect agreement with measured depths. Note that the points fall very close to the line. This means that the equation modeled the depth, Yt, fairly accurately by using a coefficient equal to unity. Points above the line represent overestimated depths.

Since Equation 3.3' was developed from a basic principle of fluid mechanics (conservation of momentum), it incorporates enough theory to account for most of the two- and three-dimensional flow effects at the gate. However, at greater depths, the calculated depths are slightly overestimated.

To correct for two- and three-dimensional flow effects, the writer decided to seek the value for the coefficient of momentum, Cm, that would minimize the average percent difference. This was accomplished through a trial and error method. For a range of 0.97 to 1.06. the average percent difference for all the data was calculated, and then the results were placed in Figure 15. The average percent difference is defined as:

$$\frac{1}{n}\Sigma\left|\frac{Ytc-Ytl}{Ytl} * 100\right|.$$

Ytc and Ytl are the calculated and measured depths, respectively. The number of data sets is represented by n.

Figure 15 indicates that Cm should equal 1.015 in order to minimize the percent difference for this data. It is recommended that other gates, such as drum or radial gates, be used in laboratory experiments to check the value of 1.015. Table 8 shows the percent differences using equation 3.3', when Cm=1.015.

By using Cm=1.015, the calculated depths are closer to the measured depths. Where Yt calc is less than Yt lab, it is shown as a negative value. To find the average percent difference for the entire experiment, all of the values were first changed to absolute values, then the average of these was found. Table 9 summarizes the information regarding the accuracy of Equation 3.3', using Cm=1.0 and 1.015. The outlier is excluded from the difference calculations.

Changing the value of Cm from 1.0 to 1.015 reduces the average, minimum, and maximum differences However, the calibration technique of finding Cm can be laborious, and the percent difference with Cm=1.0 is only 2.1%.

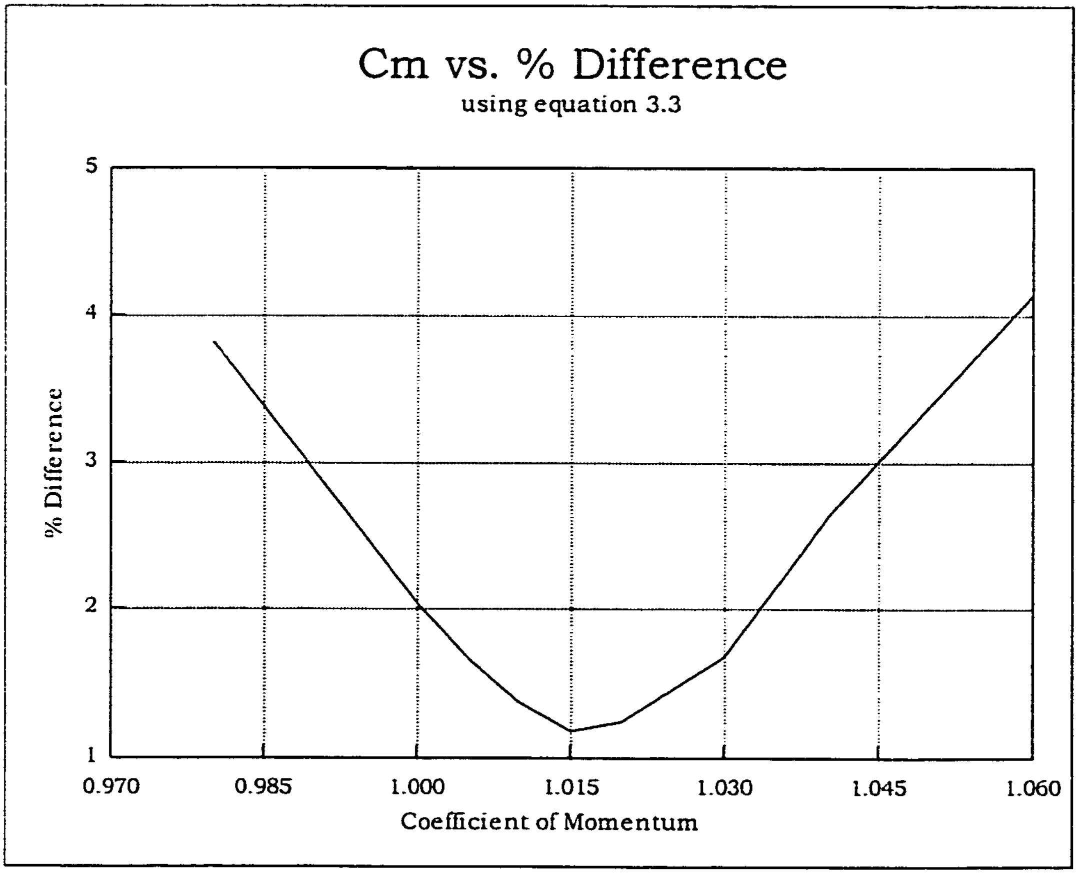

Figure 15. Finding the coefficient of momentum.

Table 8. Verification of equation 3.3' (Cm=1.015)

Yg ft	Yu ft	Yd ft	Cc ft	Yj ft	q cfs/ft	Yt lab ft	Yt calc ft (Cm=1.015)	% error 100(Ytc-Ytl) / Ytl
0.25	1.217	0.867	0.591	0.148	0.735	0.930	0.957	2.9
0.25	1.550	1.183	0.590	0.147	0.735	1.214	1.246	2.7
0.25	1.808	1.433	0.589	0.147	0.735	1.473	1.481	0.5
0.25	2.025	1.650	0.588	0.147	0.730	1.676	1.686	0.6
0.25	2.475	2.125	0.587	0.147	0.723	2.140	2.141	0.0
0.25	1.200	0.833	0.591	0.148	0.735	0.915	0.927	1.3
0.25	2.092	1.450	0.588	0.147	0.950	1.535	1.542	0.4
0.25	2.608	1.975	0.587	0.147	0.937	2.030	2.030	0.0
0.25	1.704	1.071	0.589	0.147	0.955	1.183	1.201	1.5
0.25	1.500	0.858	0.590	0.147	0.960	0.996	1.019	2.3
0.25	1.308	0.675	0.591	0.148	0.960	0.845	0.869	2.9
0.25	1.158	0.529	0.592	0.148	0.960	0.742	0.758	2.3
0.25	1.042	0.400	0.593	0.148	0.960	0.675	0.670	-0.8
0.50	0.650	0.525	0.614	0.307	0.965	0.590	0.598	1.4
0.50	0.883	0.742	0.606	0.303	0.965	0.817	0.807	-1.2
0.50	1.100	0.942	0.601	0.301	0.965	1.008	0.996	-1.2
0.50	1.425	1.267	0.597	0.299	0.965	1.308	1.305	-0.2
0.50	1.158	1.000	0.600	0.300	0.962	1.058	1.051	-0.7
0.50	1.633	1.475	0.595	0.298	0.950	1.490	1.503	0.9
0.50	0.725	0.596	0.611	0.305	0.965	0.663	0.668	0.8
0.33	1.017	0.600	0.596	0.199	1.035	0.785	0.769	-2.0
0.33	0.800	0.400	0.600	0.200	1.035	0.620	0.612	-1.3
0.33	1.142	0.725	0.595	0.198	1.035	0.878	0.874	-0.5
0.33	1.592	1.175	0.591	0.197	1.025	1.271	1.270	-0.1
0.33	1.883	1.458	0.590	0.197	1.025	1.533	1.531	-0.1
0.17	2.133	1.108	0.586	0.098	0.811	1.248	1.252	0.3
0.17	2.608	1.633	0.586	0.098	0.805	1.720	1.722	0.1
0.17	1.608	0.558	0.587	0.098	0.823	0.815	0.819	0.5
0.17	1.225	0.417	0.588	0.098	0.710	0.685	0.658	-3.9
0.17	0.583	0.175	0.594	0.099	0.523	0.404	0.394	-2.6
0.17	0.725	0.242	0.592	0.099	0.518	0.477	0.428	-10.2*
0.08	1.100	0.360	0.586	0.049	0.341	0.496	0.506	2.0
0.08	1.508	0.767	0.585	0.049	0.341	0.841	0.840	-0.1
0.08	1.725	0.850	0.585	0.049	0.374	0.910	0.930	2.2
0.08	2.467	1.617	0.584	0.049	0.374	1.642	1.645	0.2
0.08	3.508	1.017	0.584	0.049	0.640	1.190	1.221	2.6

*Outlier

Avg = 1.2%
Max = 3.9%

The accuracy of Equation 3.3' with Cm=1.015 can be seen graphically by plotting the measured depths against the calculated depths (see Figure 16). The points represent calculated depths, and they should fall directly on the 45° line if there is perfect agreement with measured depths. The points fall closer to the line, with the new value of 1.015 for the coefficient of momentum.

Table 9. Accuracy summary for equation 3.3' (Cm=1.0, 1.015)

Accuracy Information	Equation 3.3' (Cm = 1.0)	Equation 3.3' (Cm = 1.015)
Average Difference (%)	2.1	1.2
Minimum Difference (%)	0.2	0.0
Maximum Difference (%)	4.5	3.9

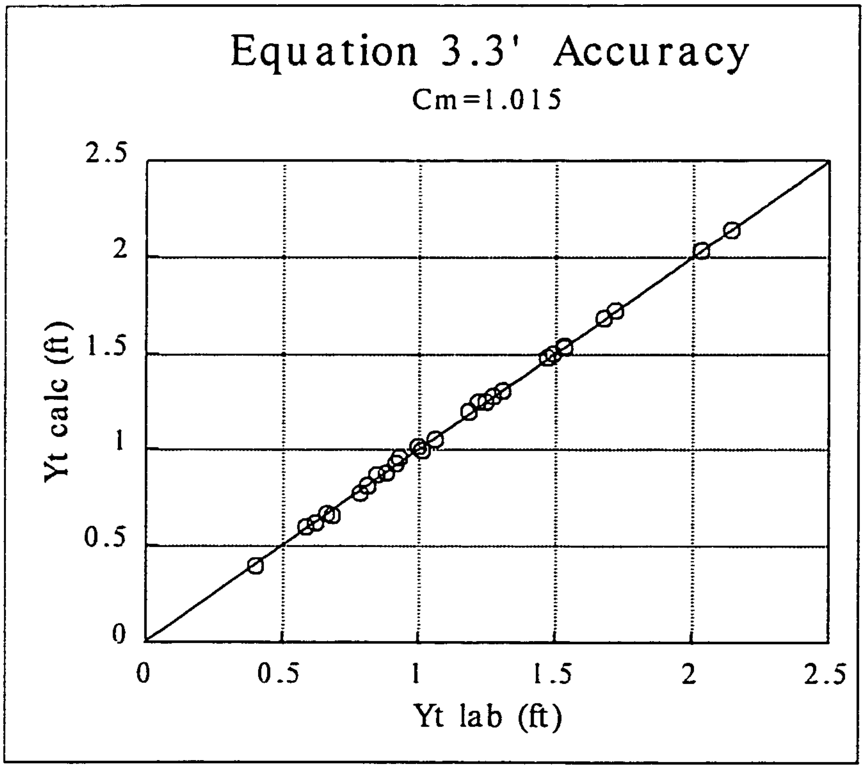

Figure 16. Accuracy of equation 3.3' (Cm=1.015).

$$\frac{C_m Y_t(Y_u^2 - Y_j{}^2)((C_m Y_t)^2 - Y_d{}^2)}{4 Y_u^2 Y_j (C_m Y_t - Y_j)} + Y_d - Y_u = 0 \tag{3.4'}$$

Equation 3.4' represents the combination of Equations 3.2 and 3.3'. It applies the principles of conservation of energy, momentum, and mass between positions 1 and 3. Its most practical application is the calculation of tailwater depth, since the upstream and downstream depths may be measured with staff gauges, and the jet depth is calculated from Yj=CcYg. This equation differs from Equation 3.4 because a "coefficient of momentum" now multiplies the tailwater depth. The coefficient of momentum, Cm, is assumed to account for two- and three-dimensional flow effects. Equation 3.4' will be verified by comparing calculated values of tailwater depth to measured values of tailwater depth.

Table 10 shows the results of the verification test using a value of 1.0 for the coefficient of momentum, and Table 11 shows the results using 1.015. The first three columns show laboratory data. The fourth column shows the coefficient of contraction. The fifth column of Table 11 shows the calculated jet depth using Yj=CcYg. The sixth column contains measured flows from Table 3. The seventh column shows the measured tailwater depth from Table 3. The eighth column shows the calculated values for tailwater depth, using Equation 3.1. The last column shows the percent difference between the calculated and measured tailwater depths.

The calculated depths agree well with the measured depths in both cases, although the use of Cm=1.015 helps a little, as in the case of Equation 3.3'. Where Yt calc is less than Yt lab, it is shown as a negative value. To find the average percent difference for

Table 10. Verification of equation 3.4' (Cm=1.0)

Yg ft	Yu ft	Yd ft	Yu-Yd ft	Cc Eqn. 3.5	Yj ft	Yt lab ft	Yt calc ft (Cm=1.0)	% error 100(Ytc-Ytl) / Ytl
0.25	1.217	0.867	0.350	0.591	0.148	0.930	0.964	3.7
0.25	1.550	1.183	0.367	0.590	0.147	1.214	1.262	3.9
0.25	1.808	1.433	0.375	0.589	0.147	1.473	1.501	1.9
0.25	2.025	1.650	0.375	0.588	0.147	1.676	1.710	2.1
0.25	2.475	2.125	0.350	0.587	0.147	2.140	2.170	1.4
0.25	1.200	0.833	0.367	0.591	0.148	0.915	0.938	2.5
0.25	2.092	1.450	0.642	0.588	0.147	1.535	1.564	1.9
0.25	2.608	1.975	0.633	0.587	0.147	2.030	2.061	1.5
0.25	1.704	1.071	0.633	0.589	0.147	1.183	1.215	2.7
0.25	1.500	0.858	0.642	0.590	0.147	0.996	1.031	3.6
0.25	1.308	0.675	0.633	0.591	0.148	0.845	0.878	3.9
0.25	1.158	0.529	0.629	0.592	0.148	0.742	0.765	3.1
0.25	1.042	0.400	0.642	0.593	0.148	0.675	0.681	0.9
0.50	0.650	0.525	0.125	0.614	0.307	0.590	0.612	3.7
0.50	0.883	0.742	0.141	0.606	0.303	0.817	0.820	0.4
0.50	1.100	0.942	0.158	0.601	0.301	1.008	1.016	0.7
0.50	1.425	1.267	0.158	0.597	0.299	1.308	1.326	1.3
0.50	1.158	1.000	0.158	0.600	0.300	1.058	1.071	1.2
0.50	1.633	1.475	0.158	0.595	0.298	1.490	1.527	2.5
0.50	0.725	0.596	0.129	0.611	0.305	0.663	0.679	2.4
0.33	1.017	0.600	0.417	0.596	0.199	0.785	0.786	0.1
0.33	0.800	0.400	0.400	0.600	0.200	0.620	0.626	1.0
0.33	1.142	0.725	0.417	0.595	0.198	0.878	0.889	1.2
0.33	1.592	1.175	0.417	0.591	0.197	1.271	1.290	1.5
0.33	1.883	1.458	0.425	0.590	0.197	1.533	1.556	1.5
0.17	2.133	1.108	1.025	0.586	0.098	1.248	1.264	1.3
0.17	2.608	1.633	0.975	0.586	0.098	1.720	1.740	1.1
0.17	1.608	0.558	1.050	0.587	0.098	0.815	0.821	0.8
0.17	1.225	0.417	0.808	0.588	0.098	0.685	0.668	-2.5
0.17	0.583	0.175	0.408	0.594	0.099	0.404	0.394	-2.5
0.17	0.725	0.242	0.483	0.592	0.099	0.477	0.459	-3.6*
0.08	1.100	0.360	0.740	0.586	0.049	0.496	0.510	3.0
0.08	1.508	0.767	0.742	0.585	0.049	0.841	0.851	1.2
0.08	1.725	0.850	0.875	0.585	0.049	0.910	0.940	3.3
0.08	2.467	1.617	0.850	0.584	0.049	1.642	1.666	1.5
0.08	3.508	1.017	2.492	0.584	0.049	1.190	1.225	2.9

*Outlier

Avg = 2.0%
Max = 3.9%

Table 11. Verification of equation 3.4' (Cm=1.015)

Yg ft	Yu ft	Yd ft	Yu-Yd ft	Cc	Yj ft	Yt lab ft	Yt calc ft (Cm=1.015)	% error 100(Ytc-Ytl)/Ytl
0.25	1.217	0.867	0.350	0.591	0.148	0.930	0.950	2.1
0.25	1.550	1.183	0.367	0.590	0.147	1.214	1.243	2.4
0.25	1.808	1.433	0.375	0.589	0.147	1.473	1.479	0.4
0.25	2.025	1.650	0.375	0.588	0.147	1.676	1.685	0.5
0.25	2.475	2.125	0.350	0.587	0.147	2.140	2.138	-0.1
0.25	1.200	0.833	0.367	0.591	0.148	0.915	0.924	1.0
0.25	2.092	1.450	0.642	0.588	0.147	1.535	1.541	0.4
0.25	2.608	1.975	0.633	0.587	0.147	2.030	2.030	0.0
0.25	1.704	1.071	0.633	0.589	0.147	1.183	1.197	1.2
0.25	1.500	0.858	0.642	0.590	0.147	0.996	1.016	2.0
0.25	1.308	0.675	0.633	0.591	0.148	0.845	0.865	2.4
0.25	1.158	0.529	0.629	0.592	0.148	0.742	0.754	1.6
0.25	1.042	0.400	0.642	0.593	0.148	0.675	0.671	-0.6
0.50	0.650	0.525	0.125	0.614	0.307	0.590	0.603	2.1
0.50	0.883	0.742	0.141	0.606	0.303	0.817	0.808	-1.1
0.50	1.100	0.942	0.158	0.601	0.301	1.008	1.001	-0.7
0.50	1.425	1.267	0.158	0.597	0.299	1.308	1.306	-0.2
0.50	1.158	1.000	0.158	0.600	0.300	1.058	1.055	-0.3
0.50	1.633	1.475	0.158	0.595	0.298	1.490	1.505	1.0
0.50	0.725	0.596	0.129	0.611	0.305	0.663	0.669	0.9
0.33	1.017	0.600	0.417	0.596	0.199	0.785	0.774	-1.4
0.33	0.800	0.400	0.400	0.600	0.200	0.620	0.617	-0.5
0.33	1.142	0.725	0.417	0.595	0.198	0.878	0.876	-0.3
0.33	1.592	1.175	0.417	0.591	0.197	1.271	1.271	0.0
0.33	1.883	1.458	0.425	0.590	0.197	1.533	1.533	0.0
0.17	2.133	1.108	1.025	0.586	0.098	1.248	1.245	-0.2
0.17	2.608	1.633	0.975	0.586	0.098	1.720	1.714	-0.4
0.17	1.608	0.558	1.050	0.587	0.098	0.815	0.809	-0.7
0.17	1.225	0.417	0.808	0.588	0.098	0.685	0.658	-3.9
0.17	0.583	0.175	0.408	0.594	0.099	0.404	0.388	-4.0
0.17	0.725	0.242	0.483	0.592	0.099	0.477	0.453	-5.0*
0.08	1.100	0.360	0.740	0.586	0.049	0.496	0.503	1.4
0.08	1.508	0.767	0.742	0.585	0.049	0.841	0.838	-0.3
0.08	1.725	0.850	0.875	0.585	0.049	0.910	0.927	1.8
0.08	2.467	1.617	0.850	0.584	0.049	1.642	1.641	0.0
0.08	3.508	1.017	2.492	0.584	0.049	1.190	1.206	1.4

*Outlier

Avg = 1.1%
Max = 4.0%

both experiments, all values were changed to absolute values, and then the average of these was calculated.

Table 12 summarizes the accuracy information for Equation 3.4'. It shows how much Cm reduced the average percent difference when it was changed from 1.0 to 1.015. The outlier is excluded from the difference calculations. By using Cm=1.015, the percent difference was reduced. However, the calibration technique of finding Cm can be laborious, and the percent difference with Cm=1.0 is still only 2.0%.

The accuracy of Equation 3.4' can be seen graphically, by plotting the measured depths against the calculated depths. Figure 17 shows how well the equation predicts the depth Yt when a coefficient is not used (or equal to unity), and Figure 18 shows the increase in accuracy when a coefficient of momentum of 1.015 is used. The points represent calculated depths, and should fall directly on the 45° lines if they agree perfectly with measured values. The points fall slightly above the line for Cm=1.0, and closer to the line for Cm=1.015. This is the same behavior as Equation 3.3', which makes sense because Equation 3.4' is a combination of Equations 3.2 and 3.3', and Equation 3.2 data was fairly symmetric around the line.

Since Equation 3.4' was developed from the basic principles of fluid mechanics, it incorporates enough theory to account for two- and three-dimensional flow effects at the gate. However, it can be improved slightly by multiplying Yt by the coefficient of momentum. It is suggested that the value of 1.015 be verified with another type of gate, such as the radial or drum.

Table 12. Accuracy summary for equation 3.4' (Cm=1.0, 1.015)

Accuracy Information	Equation 3.4' (Cm=1.0)	Equation 3.4' (Cm=1.015)
Average Difference (%)	2.0	1.1
Minimum Difference (%)	0.1	0.0
Maximum Difference (%)	3.9	4.0

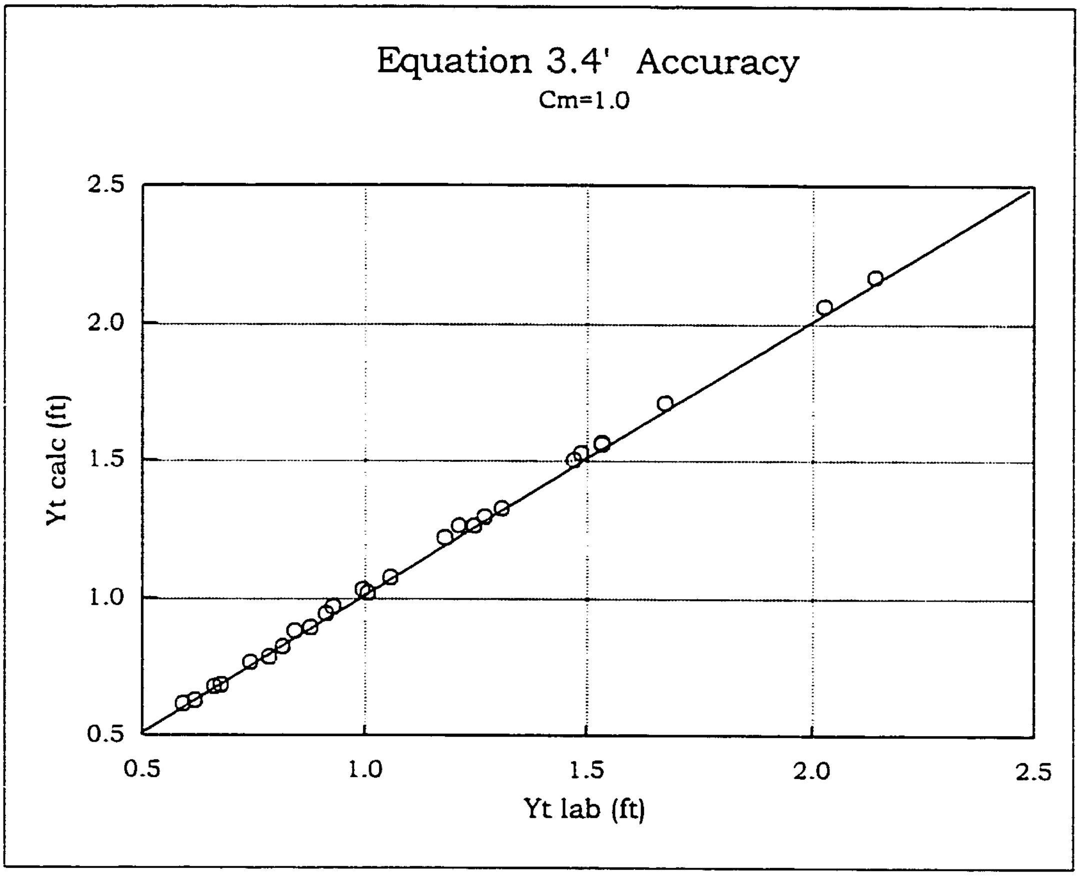

Figure 17. Accuracy of equation 3.4' (Cm=1.0).

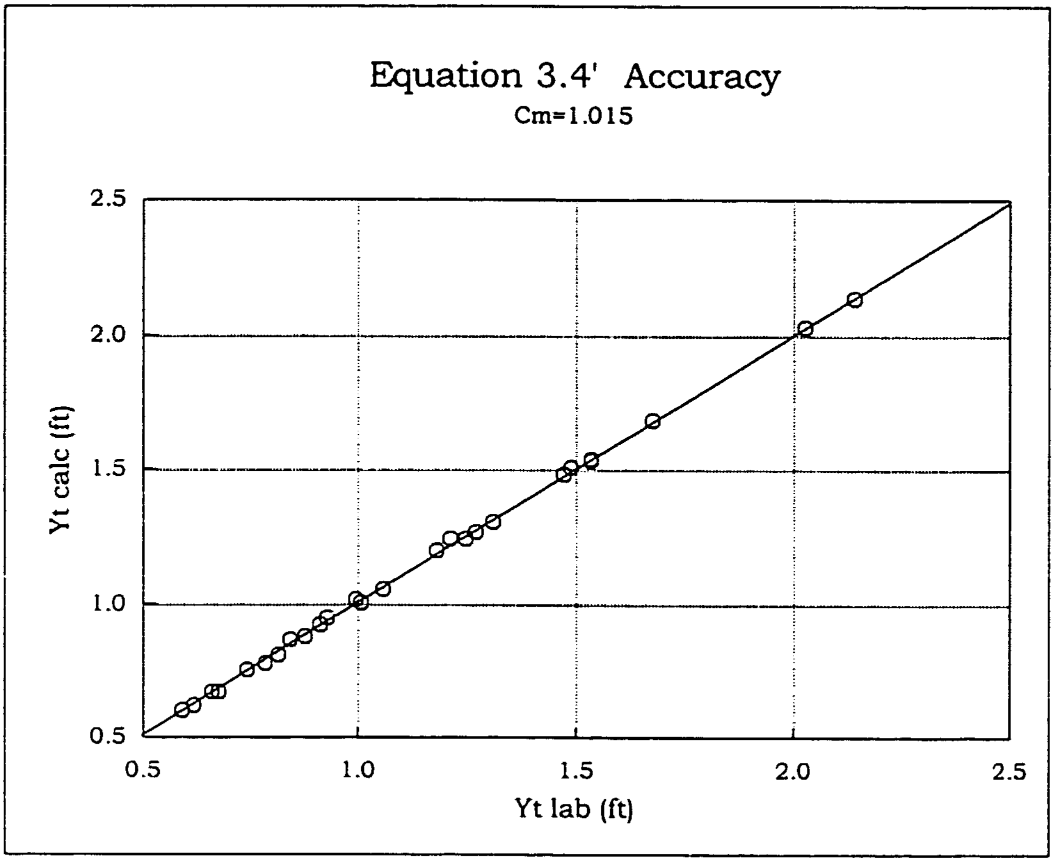

Figure 18. Accuracy of equation 3.4' (Cm=1.015).

CHAPTER VI
EXAMPLE PROBLEMS

Example Problem 1

Determine whether the flow from under the gate is free or submerged.

Solution:

Step 1: Assuming Yu and q are known, solve for Yj using equation 3.1.

Step 2: Compute the momentum associated with Yj and compare it to the computed momentum associated with Yt. If the momentum for Yt is greater than the momentum for Yj, then the condition will be submerged flow.

Example Problem 2

Find the flow at a given Yu, Yg, Yt.

Solution:

Step 1: Find Yj from Equation 3.6.

Step 2: Find Yd from Equation 3.4.

Step 4: Find q from Equation 3.2.

Example Problem 3

Find the flow at a given Yu, Yg, Yd.

Solution:

Step 1: Find Yj from Equation 3.6.

Step 2: Find q from Equation 3.2.

Example Problem 4

Predict the upstream depth at a given Yg, Yt and q.

Solution:

Step 1: Find Yu, Yj, and Yd simultaneously from Equations 3.2, 3.3, and 3.6.

Example Problem 5

Find the upstream depth at a given q, Yt, Yd

Solution:

Step 1: Find Yj from Equation 3.3.

Step 2: Find Yu from Equation 3.2 or Equation 3.4.

Example Problem 6

Find the necessary gate opening to pass a certain flow at a given Yu and Yd.

Solution:

Step 1: Find Yj from Equation 3.2.

Step 2: Find Yg from Equation 3.6.

Example Problem 7

Find the necessary gate opening to pass a certain flow at a given Yu and Yt.

Solution:

Step 1: Find Yg, Yj,Yd simultaneously from Equations 3.2, 3.6, and 3.3.

or

Find Yg, Yj,Yd simultaneously from Equations 3.2, 3.6, and 3.4.

Example Problem 8

Find the tailwater depth at a given Yu, Yg, q.

Solution:

Step 1: Find Yj, Yd, Yt simultaneously from Equations 3.2, 3.3, and 3.6.

Example Problem 9

Find the tailwater depth at a given Yu, Yg, Yd.

Solution:

Step 1: Find Yj from Equation 3.6.

Step 2: Find Yd from Equation 3.2.

Step 3: Find Yt from Equation 3.3 or 3.4.

CHAPTER VII

COMPUTER PROGRAM

Program Overview

To implement the equations discussed in this report, the writer developed a computer program called SUBGATE, using the FORTRAN computer language. This program solves three individual problems dealing with submerged flow at a vertical (sluice) sharp-edged gate. All three problems have a common condition, which is a reservoir supplying a reach of open channel, with a gate downstream. Downstream of the gate, there is a second reach which may end in different ways.

The first downstream condition includes a long channel downstream of the gate, resulting in "normal" flow. The second condition involves a free overfall, which results in "critical" flow. The third condition involves a downstream reservoir. Throughout the remainder of this thesis, these problems will be referred to as Norm, Over, and Res, respectively.

The input file for the program is titled SUBGATE.IN, and it holds the data which define the unchanging parameters of the system, i.e., length, base width, bed slope, side slope, Manning's n, acceleration due to gravity, and loss coefficients for the reservoir(s). These parameters may be different for the two reaches, but cannot vary within a reach. The output file is titled SUBGATE.OUT, which gives the user a history of all output during the program run time, which may include several runs.

Newton-Raphson Method

The computer program SUBGATE.FOR contains 563 lines of code, although the core is found in 10 lines which implement the Newton method. This algorithm is adapted to solving systems of non-linear equations, which is ideal for the submerged gate problem. Variables are solved itteratively by improving and comparing to previous values. The solution is found when the error criteria has been met. Line 23, in the program code, sets the error to 0.001. The user may wish to change this value and recompile the program.

Some methods that solve systems of equations have linear convergence, whereas the Newton-Raphson method converges quadratically, which enables it to solve more rapidly. However, since there are usually multiple real roots to nonlinear equations, an error can occur if the guesses are not somewhat close to the real values. SUBGATE gives the user the option of estimating the initial guesses for the variables, or allowing the computer do it. In most cases, the computer does a reasonable job, but if the solver crashes, the user will have to use intuition to do a better job of estimating.

Problems

The first problem is called NORM, which is an abbreviation of the word normal. In this case, the downstream condition is a long, undisturbed length of channel enabling the flow to reach its normal state. This normal flow condition is described by flow and depth. NORM is the fastest problem to solve for two reasons.

First, the normal depth, Yt, is calculated from Manning's equation. Second, there are five unknowns and five equations instead of six unknowns and six equations, as in the other two problems. Solve time ranges from 1 to 5 seconds depending on computer speed and accuracy of initial estimates. Figure 19 shows the parameters in the problem.

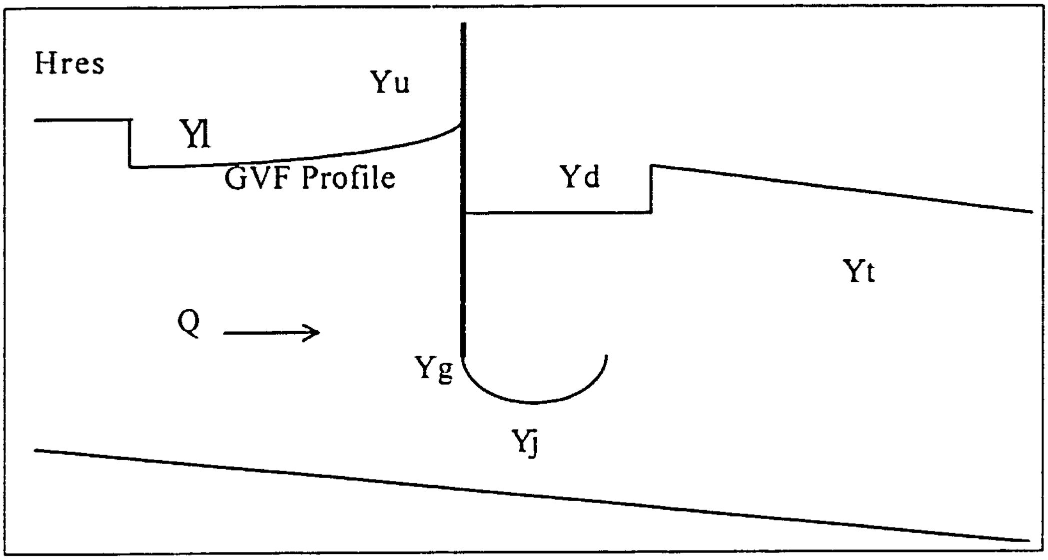

Figure 19. Normal flow problem (Norm).

The second problem is called OVER, which is an abbreviation of the word overfall. In this case, the second reach of channel ends in the critical flow condition with a Froude number equal to unity. Examples of this would be a waterfall, or a sudden drop in bed elevation. Under critical conditions, the flow will be a maximum for a given upstream reservoir head and gate setting. Using the continuity principle, this flow will be the same through the entire channel. The depth at the overfall is the critical depth, Yc, which is calculated using the critical flow equation. This equation replaces the Manning's equation in the previous problem.

Since there are now six variables instead of five, an additional equation is introduced. This equation is the ordinary differential equation that describes the GVF profile. It requires more computer effort so the solve time is greater. Depending on computer speed and initial estimates, the solution takes 4-15 seconds. Figure 20 shows the parameters in the problem.

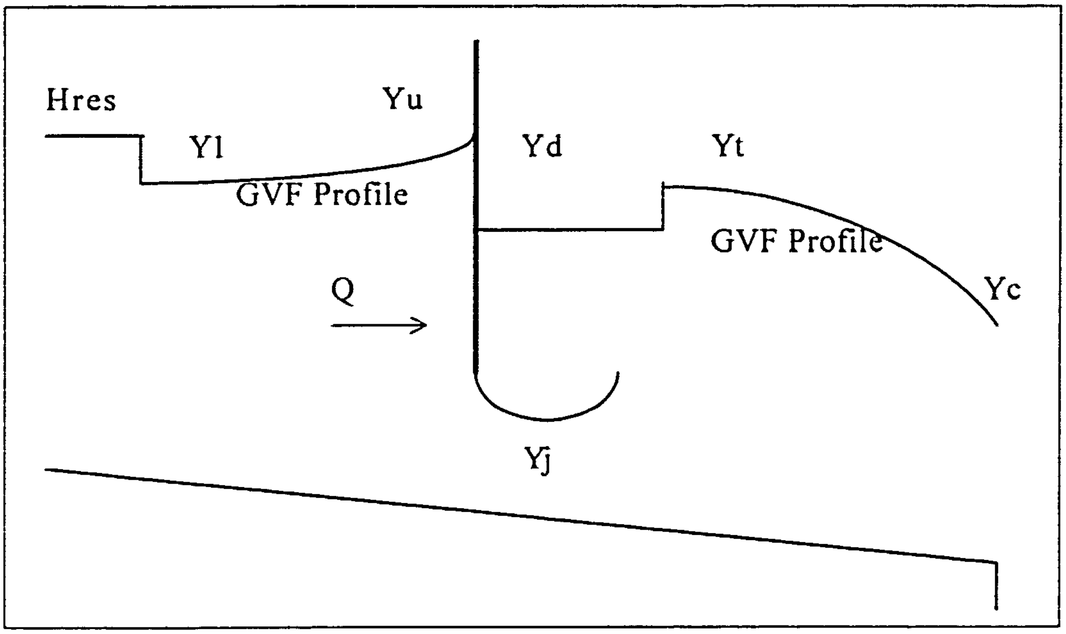

Figure 20. Overfall problem (OVER).

The third problem is called RES, which is an abbreviation of the word reservoir. In this case, the second reach ends with a reservoir. Similar to the previous problem, Yend is involved. Instead of critical depth, it is the depth in the channel just before the flow enters the second reservoir. Yend will be slightly lower than H2, the height of the second reservoir above the channel bottom. This depth is calculated using specific energy balance from the channel to the second reservoir, with the entire velocity head being dissipated.

As in the overfall problem, this depth is tied back to depth Yt through a GVF profile. This problem is the most unforgiving on bad initial estimates. Solve time ranges from 4-15 seconds, depending on computer speed and initial estimates. Figure 21 shows the parameters in the problem.

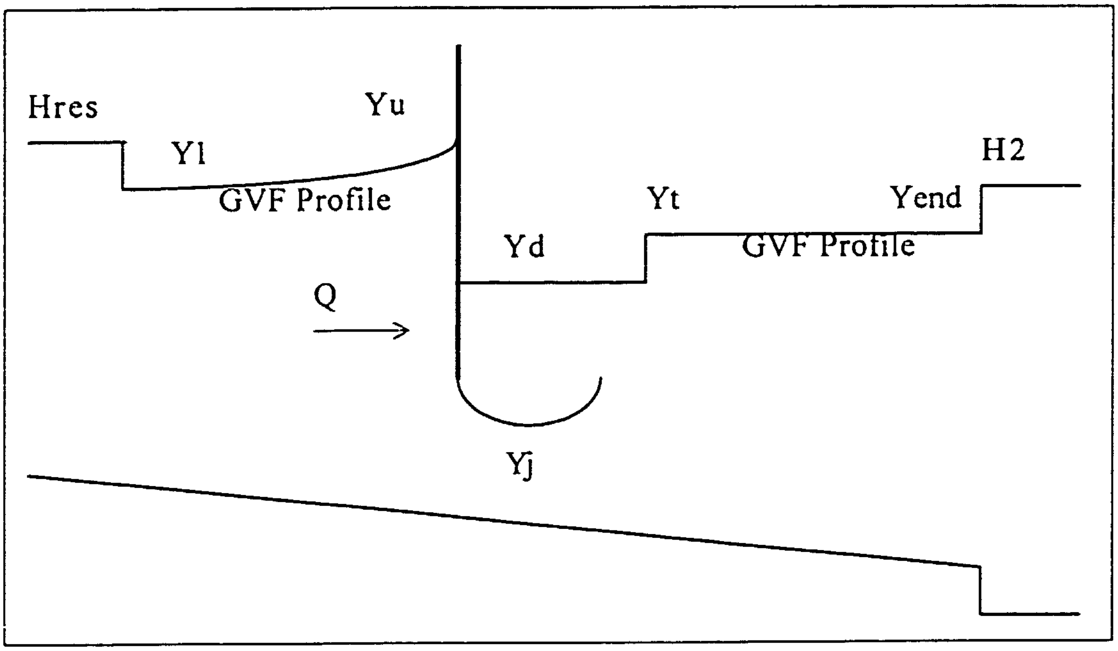

Figure 21. Reservoir problem (RES).

Variables

Y1 is the depth at the beginning of the channel, immediately downstream of the reservoir. Since depth contributes more to specific energy than does velocity head, the computer's initial estimate is Y1 = 0.9H, where H is the reservoir water surface above the channel bottom. This means that most of the reservoir head, H, is converted into channel depth and only 10% is converted into velocity head. This assumption is best for mild channels, but for steeper slopes the velocity head will increase and the depth will decrease, changing the energy makeup of the system and the validity of the assumption.

Yu is the depth immediately upstream of the gate. It should be measured about 5 feet away, before the water surface curves up to a stagnation point on the gate. Yu and Y1 are at the downstream and upstream ends of a gradually varied flow (GVF) profile, respectively. In the direction of flow, the water surface will slope up (M1 curve) for lower gate openings and slope down (M2 curve) for higher gate openings. Since Yu may be greater or less than Y1, the computer's initial estimate is Yu=Y1.

Yd is the depth immediately downstream of the gate. Under free flow conditions, this depth would be the jet emerging from beneath the gate; but under submerged conditions, it is the submergence depth which includes the jet depth and the water swirling above it. Yd is more difficult to estimate because it can vary widely, ranging from slightly more than Yj to almost Yu. Since the only certainty is that it is greater than the jet depth, the computer's initial guess is Yd = 1.5Yj .

Yt is the depth downstream of Yd. It is likely to occur around 10 feet downstream of the gate, and is preceded by an abrupt rise in the water surface. In the case of free flow, the jet is in a supercritical state and dissipates through a hydraulic jump, ending in a subcritical condition with greater depth and lower velocity.

A similar situation occurs during submerged flow, where the submerged jet dissipates its energy through an increase in depth, downstream of the gate. Since the downstream submergence depth, Yd, may range between the jet depth and the upstream depth, the difference between Yd and Yt also has a range of possiblilities.

Since the only certainty is that Yt is greater than Yd, the program's initial estimate is Yt = 1.5Yd. In the first problem, involving normal flow, Yt is calculated using Manning's equation. For the other two problems, it is solved using a GVF profile from the downstream boundary condition and moving upstream.

Yend is used in the problems RES and OVER. As the name implies, it is the final depth in the channel. In the problem involving a downstream reservoir, Yend is calculated using energy balance from the channel to the reservoir, and is tied back to Yt through a GVF profile. This depth will be slightly less than the depth of the reservoir above the channel bottom, so assume Yend = .9H2 (90% of the vertical difference between the reservoir surface elevation and the channel bottom). In the problem involving a sudden drop in the channel bed, Yend is the critical depth, which is calculated from the critical flow equation and is tied back to Yt through a GVF curve. The computer's estimate is that Yend = 1 foot or 0.3 meters.

Q is the steady-state volumetric flow. Because of the continuity principle, it is constant throughout the entire channel, and is found in every equation. Flow has a much greater range of values than depth, but is also the most forgiving on bad guesses. A good initial estimate seems to be Q = 20H (20 times the reservoir head above the channel bottom). These suggestions for initial guesses are very general, so if the user is familiar with open channel hydraulics, he/she should be able to supply more reasonable values. Nevertheless, the above initial estimates may be used in the program by selecting the computer guess option.

Equations

Table 13 summarizes the equations used in the SUBGATE computer program. A description of each equation is followed by the problem(s) it pertains to, and the unknown variable it solves for. NORM uses five equations to solve for five unknowns, whereas OVER and RES use six equations to solve for six unknowns. Jet depth is solved from Equation 3.6 immediately before it is used in the solver routine.

Table 13. Equations used in the computer program

Equation	Problem Type	Variable
Energy balance at the reservoir $H - Y_1 - \frac{Q^2}{2g(b_1 Y_1 + m_1 Y_1^2)^2} = 0$	Norm, Over, Res	Y1
Special specific energy between positions 1 and 2 $Y_u - Y_d + \frac{Q^2}{2g}(\frac{1}{(b_1 Y_u + m_1 Y_u^2)^2} - \frac{1}{(b_g Y_j)^2}) = 0$	Norm, Over, Res	Yu
Special momentum between positions 2 and 3 $\frac{1}{2}b_2(Y_d^2 - Y_t{}^2) + \frac{1}{3}m_2(Y_d^3 - Y_t)^3 + \frac{Q^2}{g}(\frac{1}{(b_2 Y_t + m_2 Y_t^2)} - \frac{1}{(b_g Y_j)}) = 0$	Norm, Over, Res	Q
GVF profile solver upstream of gate $Y_1 - Y_1(ode - Y_u) = 0$	Norm, Over, Res	Yd
GVF profile solver downstream of gate $Y_t - Y_t(ode - Y_{end}) = 0$	Over, Res	Yt
Manning's formula for the normal flow condition $\frac{Qn}{C} - \frac{(b_2 Y_t + m_2 Y_t^2)^{5/3}}{(b_2 + 2Y_t\sqrt{m_2^2+1})^{2/3}} = 0$	Norm	Yt
Energy balance at downstream reservoir $H_2 - Y_{end} - \frac{Q^2}{2g(b_2 Y_{end} + m_2 Y_{end}^2)^2} = 0$	Res	Yend
Critical depth energy equation for overfall $g(b_2 Y_{end} + m_2 Y_{end}^2)^3 - Q^2(b_2 + 2m_2 Y_{end}) = 0$	Over	Yend

Example NORM

In this example problem, the downstream reach allows the flow to reach normal conditions a short distance downstream of the gate. The channel is trapezoidal, with a bottom width of 10 feet and a side slope of 2 (2H:1V). The gate width is 10 feet, and the lengths of the upstream and downstream reaches are 2000 feet and 3000 feet, respectively. Manning's roughness, n, is equal to 0.015 both upstream and downstream. The upstream and downstream slopes are .0002 and .0004, respectively. The acceleration of gravity is assumed to be 32.2 ft^2/sec. The entrance head loss coefficient from the upstream reservoir to the channel is equal to .05. The flow and momentum coefficients are equal to 1. and 1.015, respectively. Figures 22, 23, and 24 show the input file, program run, and output file, respectively.

```
10. 10. 10. 2000. 3000. .015 .015 2. 2. .0002 .0004 32.2 1. .05 1. 1.015
INPUT:
Gate Width       bg
Bottom Width     b1
Bottom Width     b2
Length           L1
Length           L2
Manning's n      FN1
Manning's n      FN2
Side Slope       FM1
Side Slope       FM2
Bed Slope        So1
Bed Slope        So2
Gravity Accl     g
Loss Coeff       KL1 (2nd reservoir problem only)
Loss Coeff       Ke1  (Entrance of channel)
Flow Coef        Cd
Momentum Coef    Cm
```

Figure 22. Input file for normal flow (NORM) example.

```
                                        11
               Q-->                     11
 ___                                    11
    \___________________________________11
                                        11
                                        11        _______________________
                                        11______/
                                        11
H      Y1                  Yu           11   Yd     Yt                Yend
                                        1
                                         \____
                                      Yg   Yj
=========================================================================
                         Input File is SubGate.in
                         Output file is SubGate.out
                         CTRL-C to quit

                    What is the downstream condition?

                         1:   Long channel
                         2:   Free overfall
                         3:   2nd reservoir
1
                 Please give reservoir head and gate opening
4
1
                         1:   computer guesses
                         2:   user guesses
1

     Yg       Cc       Q        Y1       Yu       Yj       Yd       Yt
    1.000    .591    71.768    3.949    4.837    .591     2.547    3.257

Upstream M1 profile

Iterations=   4
Error=   .00031434
```

Figure 23. Program run for normal flow (NORM) example.

```
                              11
      Q-->                    11
---\                          11
    \_________________________11
                              11
                              11
                              11_______/ ___________________
                              11
H     Y1              Yu      11  Yd       Yt                Yend
                              1
                               \_____
                             Yg   Yj
=====================================================================
```

This solution is for a long channel downstream
Upstream reservoir head is 4.00

Yg	Cc	Q	Y1	Yu	Yj	Yd	Yt
1.000	.591	71.768	3.949	4.837	.591	2.547	3.257

Figure 24. Output file for normal flow (NORM) example.

Example OVER

In this problem, there is a sudden drop in the bed elevation, or a complete drop off, such as a waterfall. To compare how the flow is affected, the same input file will be used. In other words, the geometry and physical characteristics of the system remain the same. Gravity acceleration, entrance losses, flow and momentum coefficients are also assumed to be the same. Notice that the flow increases and the depths decrease, when the downstream condition changes from normal flow to critical flow. Figures 25, 26, and 27 show the input file, the program run, and the output file, respectively.

```
10. 10. 10. 2000. 3000. .015 .015 2. 2. .0002 .0004 32.2 1. .05 1. 1.015
INPUT:
Gate Width       bg
Bottom Width     b1
Bottom Width     b2
Length           L1
Length           L2
Manning's n      FN1
Manning's n      FN2
Side Slope       FM1
Side Slope       FM2
Bed Slope        So1
Bed Slope        So2
Gravity Accl     g
Loss Coeff       KL1 (2nd reservoir problem only)
Loss Coeff       Ke1  (Entrance of channel)
Flow Coef        Cd
Momentum Coef    Cm
```

Figure 25. Input file for critical flow (OVER) example.

```
                                   11
___          Q-->                  11
   \_______________________________11
                                   11
                                   11          ______________________
                                   11______/
                                   11
H     Y1                    Yu     11   Yd     Yt               Yend
                                   I
                                    \_____
                                  Yg   Yj
==========================================================================

                       Input File is SubGate.in
                       Output file is SubGate.out
                            CTRL-C to quit

                  What is the downstream condition?

                         1:   Long channel
                         2:   Free overfall
                         3:   2nd reservoir
2
             Please give reservoir head and gate opening
4
1
                         1:  computer guesses
                         2:  user guesses
1

   Yg      Cc      Q       Y1      Yu      Yj      Yd      Yt      Yend
  1.000   .591   73.715   3.946   4.782   .591    2.366   3.154   1.191

Upstream M1 profile
Downstream M2 profile

Iterations=  14
Error=  .00047021
```

Figure 26. Program run for critical flow (OVER) example.

```
                                    11
___         Q-->                    11
   \________________________________11
                                    11
                                    11
                                    11_______  /  ___________________________
                                    11
H      Y1                    Yu     11  Yd      Yt                  Yend
                                    1
                                     \_____
                                  Yg   Yj
==========================================================================
```

This solution is for a free overfall downstream
Upstream reservoir head is 4.00

Yg	Cc	Q	Y1	Yu	Yj	Yd	Yt	Yend
1.000	.591	73.715	3.946	4.782	.591	2.366	3.154	1.191

Figure 27. Output file for critical flow (OVER) example.

Example RES

In this problem, the downstream reach discharges into a reservoir. To see how the flow conditions change based upon boundary condition, the same input file will be used again. In other words, the geometry and physical characteristics of the system remain the same. Gravity acceleration, entrance losses, and flow and momentum coefficients are also assumed to be the same. Notice that if the downstream reservoir head is the same as the upstream reservoir head, this problem yields the lowest flow and the highest depths of the three problem types.

```
10. 10. 10. 2000. 3000. .015 .015 2. 2. .0002 .0004 32.2 1. .05 1. 1.015
INPUT:
Gate Width      bg
Bottom Width    b1
Bottom Width    b2
Length          L1
Length          L2
Manning's n     FN1
Manning's n     FN2
Side Slope      FM1
Side Slope      FM2
Bed Slope       So1
Bed Slope       So2
Gravity Accl    g
Loss Coeff      KL1 (2nd reservoir problem only)
Loss Coeff      Ke1  (Entrance of channel)
Flow Coef       Cd
Momentum Coef   Cm
```

Figure 28. Input file for reservoir (RES) example.

```
                                    11
___          Q-->                   11
   \________________________________11
                                    11
                                    11
                                    11_______/ ________________________
                                    11
H     Yl                       Yu   11   Yd      Yt                 Yend
                                    1
                                     \_____
                                  Yg   Yj
=========================================================================
```

Input File is SubGate.in
Output file is SubGate.out
CTRL-C to quit

What is the downstream condition?

1: Long channel
2: Free overfall
3: 2nd reservoir

3

Please give reservoir head and gate opening

4
1

Give 2nd reservoir head above channel bottom

4

1: computer guesses
2: user guesses

1

Yg	Cc	Q	Y1	Yu	Yj	Yd	Yt	Yend
1.000	.591	68.770	3.953	4.917	.591	2.813	3.418	3.953

Upstream M1 profile
Downstream M1 profile
Iterations= 4
Error= .00008997

Figure 29. Program run for reservoir (RES) example.

```
                                  11
__          Q-->                  11
  \_______________________________11
                                  11
                                  11                ______________________
                                  11_______/
                                  11
H      Y1                   Yu    11   Yd     Yt                  Yend
                                  1
                                   \_____
                                Yg   Yj
================================================================================
```

This solution is for a second reservoir downstream
Upstream reservoir head is 4.00
Downstream reservoir head is 4.00

Yg	Cc	Q	Y1	Yu	Yj	Yd	Yt	Yend
1.000	.591	68.770	3.953	4.917	.591	2.813	3.418	3.953

Figure 30. Output file for reservoir (RES) example.

CHAPTER VIII

SUMMARY AND RECOMMENDATIONS

Two types of flow may exist at an open channel gate: free and submerged. In free flow, the gate creates a subcritical upstream depth, a supercritical jet depth, and, if the channel continues with a mild slope, a tailwater depth which is preceded by a hydraulic jump. In submerged flow, the momentum function associated with the tailwater depth becomes greater than the momentum function associated with the jet, which causes the jet to become submerged under a swirling roller. Submerged flow is controlled by the upstream depth, the gate opening, and the downstream submergence depth, whereas free flow is only controlled by the upstream depth and the gate opening.

The focus of this study is limited to submerged flow. Free flow is examined, but only as a precursor to submerged flow. For example, the special energy and momentum equations were developed for free flow, then modified for submerged flow. For a comprehensive understanding of free flow and the equations used to solve it, the reader should find other literature.

Gates may be used for control and measurement in open channel flow. The equations that predict flow typically contain coefficients that are determined by field calibration. The use of these coefficients has been necessary since the equations are simple in their structure and are one-dimensional, which neglects two- and three-dimensional flow effects.

This study developed a special specific energy equation and a special momentum equation which predict flow and depth for submerged flow. When used without correction coefficients, the results are very reasonable, although a coefficient of momentum slightly improves the error for the special momentum equation.

The equations were developed from basic principles of fluid mechanics, i.e., conservation of energy, momentum, and mass. Instead of reducing the complexity of the equations (which increases the need for correction coefficients), the mathematics were carried through to yield sound formulas that are grounded in fundamental principles. The special energy and momentum equations are easily implemented with a programmable calculator or computer.

The special specific energy equation can be used to predict submerged flow. The development of the equation consists of equating the upstream specific energy to the downstream "special" specific energy. The upstream specific energy is the sum of the upstream depth and the upstream velocity head. The downstream special specific energy is the sum of the downstream submergence depth (consisting of the jet depth and the overlying roller) and the velocity head of the jet. Comparison of computed and measured flows show agreement to within 1.6%, without a correction coefficient.

The special momentum equation is used to model the flow downstream of the gate, where the jet dissipates and the water surface rises from the submergence depth to the tailwater depth. This development of the special momentum equation is accomplished by equating the special momentum function (with the submergence depth in the hydrostatic term and jet depth in the dynamic term) to the momentum function associated with the tailwater depth. A coefficient of momentum multiplies the tailwater depth, which accounts for two- and three-dimensional flow effects. Using a value of 1.0 for the coefficient gives agreement to within 2.1% between computed and measured depths. With a coefficient equal to 1.015, the difference between computed and measured depths decreases to 1.2%.

The "combination" equation is created when the special specific energy and special momentum equations are combined, which eliminates flow as a variable. Therefore, only depths are involved in the combination equation. When the equation is used to predict tailwater depth, with the other depths known, there is agreement to within 2.0% using a coefficient of 1.0, and to within 1.1% with a coefficient of 1.015. The coefficient is the same coefficient of momentum that is used in the special momentum equation.

The equations were developed assuming a rectangular cross section. Then, the geometry in the equations was modified to allow for trapezoidal or rectangular cross sections. For a rectangular cross section, the side slope, m, is zero. For a trapezoidal cross section, the side slope is a value greater than zero.

Although a vertical (sluice) gate is referred to throughout the study, it is expected that the equations may be used for other types of gates, as long as the conditions are similar. These conditions are that the fluid emerges from the bottom of the gate as a jet that flows under the downstream flow with an overriding roller, and that submergence depth is less than tailwater depth, a short distance downstream (Jeppson, 1994).

It is also expected that the format of the contraction coefficient (Cc) equation (Cc=a+b*gate opening/upstream depth) will be the same for other gate types, where a and b are experimentally determined constants for each gate type. The constant, a, is the lower contraction limit which represents the maximum amount of contraction. The constant, b, is the multiplier of the ratio, and defines the shape of the equation's curve. For the vertical (sluice) gate, the values for 'a' and 'b' are 0.583 and 0.04, respecively.

How accurate an equation duplicated measured values was investigated by calculating one variable, while using measured values for the other variables in the equation. The writer's goal was for agreement within 5% of past reported measured values, or a "percent difference" of 5%. The special specific energy equation predicted flow to within 1.6% of measured values without a correction coefficient. The special momentum equation calculated tailwater depth to within 2.0% of measured values without a coefficient, and to within 1.2% using a coefficient of 1.015. The combination equation calculated tailwater depth to within 2.0% of measured values without a coefficient, and to within 1.1% using a coefficient of 1.015. Therefore, the goal of reaching an average difference of 5% was achieved and surpassed.

A FORTRAN computer program called "SUBGATE," was developed to implement the equations for the case of submerged flow, and it solves three types of problems. All of the problems have an upstream boundary condition: a reservoir supplying a reach of open channel. The problems differ because of downstream boundary conditions: (1) a downstream uniform, or normal depth, (2) an overfall which produces critical flow, or (3) a downstream reservoir.

The program reads an input file (SUBGATE.IN) and writes an output file (SUBGATE.OUT). The physical characteristics of the channel and gate are stored in the input file, so the user is only prompted for the upstream and downstream reservoir heads and the gate opening during the program run. The output consists of the description of three characteristics: the flow, the key depths, and the gradually varied flow (GVF) profiles. It also displays the number of iterations and error in the final solution.

The core of the computer program contains the code which implements the Newton-Raphson Method, which solves a system of nonlinear equations. This method requires that initial estimates be made for all variables in the equations to be solved. The user has the option of making the estimates or allowing the computer to do it. The estimates which the computer makes are usually good enough for convergence to occur, but the user may have to estimate the variables if the program crashes.

Some of the limitations of this research are: (1) The equations are limited to submerged flow, and do not model free flow. (2) The value for the coefficient of momentum was found through trial and error with the data referred to in the report. It is recommended that someone verify the value of 1.015 with other data. The coefficient may vary depending on the gate type, or some other parameter. (3) The data in this report are only for a vertical (sluice) sharp-edged gate, and the equations were verified for that gate type. It is recommended that someone test these equations with data from other types of gates.

LITERATURE CITED

Boiten, Wubbo. 1992. Vertical gates for distribution of irrigation water. Delft Hydraulics and Agricultural University, Wageningen, Netherlands. 15 p.

Dirkzwager, M. 1958. Formulas for undershotflow of gates. Delft Hydraulics and Agricultural University. Unpublished note N2. 6 p.

Garbrecht, G. 1980. Measurement of flow rate by means of plane sluice gates. ICID Bulletin 29(1): 1-11.

Henderson, F.M. 1966. Open channel flow. The Macmillan Company, New York. 544 p.

Henry, H.R. 1950. A study of flow from a submerged sluice gate. Unpublished MS thesis. State University of Iowa Library. Iowa City, Iowa. 100 p.

Kolkman, P.A. 1989. Discharge relations for hydraulic structures and head losses from different components. Delft Hydraulics and Agricultural University. Report No. Q953. 31 p.

Jeppson, R.W. 1994. Open channel flow. Civil & Environmental Engineering Department, Utah State University, Logan, Utah. 451 p.

Rajaratnam, N. and K. Subramanya. 1967. Flow equation for the sluice gate. Journal of the Irrigation and Drainage Division. September: 167-181.

Woycicki, K. 1934. The hydraulic jump and its top roll and the discharge of sluice gates. U.S. Bureau of Reclamation, Technical Memorandum 435. 10 p.

APPENDIX

SUBGATE.FOR Computer Code

```
CC      Program SubGate solves flow and depth associated submerged flow.
CC      All options solve energy at channel entrance, GVF upstream and downstream, energy at the gate,
CC      and momentum downstream of the gate. PROBLEM 1 solves a long channel downstream
CC      substituting Manning's equation in place of the downstream GVF.
CC      PROBLEM 2 solves a free overfall downstream. An additional depth Yend occurs at the overfall.
CC      It is the critical depth and is solved by the critical flow eqn. The downstream GVF begins here and
CC      moves upstream. PROBLEM 3 solves a second reservoir downstream. Another depth
CC      Yend occurs just before the channel empties into reservoir and CC is solved using energy from the
CC      channel to the second reservoir.
        INTEGER*2 INDX(6)
        LOGICAL*2 RES,NORM,OVER,FREE
        REAL F(6),D(6,6),X(6),KE1,KL1,XG(6)
        COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,NORM,OVER,RES
        COMMON /TRAS/H,G,G2,Yg,C,QN,KE1,KL1,X,CD1,H2,CM
        COMMON /GEO/bg,b1,b2,So1,So2,FN1,FN2,FM1,FM2,FL1,FL2
        EQUIVALENCE(Q,X(1)),(Y1,X(2)),(Yu,X(3)),(Yd,X(4)),(Yt,X(5)),(Yend,X(6))
        OPEN(FILE='SubGate.in',STATUS='OLD',UNIT=3)
        OPEN(FILE='SubGate.out',STATUS='UNKNOWN',UNIT=2)
        READ(3,*) BG,B1,B2,FL1,FL2,FN1,FN2,FM1,FM2,SO1,SO2,G,KL1,KE1,Cd,Cm
        ERR=.001
114     NWRITE=1
        WRITE(*,300)
        WRITE(2,300)
300     FORMAT(34X,'Program'/31X,'S U B G A T E'//37X,'I I'/'____',1X,
   &    'Q-->',27X,'I I'/5X,'\______________________________I I'/37X,'I I'
   &    /37X,'I I',8X,'_____________________________'/37X,'I I______/'/
   &    37X,'I I'/2X,'H',5X,'Y1',22X,'Yu',3X,'I I',2X,'Yd',8X,'Yt',15X,'
   &    Yend'/37X,'I'/38X,'\_____'/37X,'Yg  Yj'/'========================
   &    ================================================')
        WRITE(2,*) ''
        WRITE(*,301)
301     FORMAT(26X,'Input file is SubGate.in'/24X,'Output file specified b
   &    y user'/31X,'CTRL-C to quit'//22X,'What is the downstream conditio
   &    n?'//30X,'1:  Long channel'/30X,'2:  Free overfall'/30X,
   &    '3:  2nd Reservoir')
        READ(*,*) PTYPE
        WRITE(*,302)
302     FORMAT(18X,'Please give reservoir head and gate opening')
        READ(*,*) H,Yg
        NORM=.FALSE.
        OVER=.FALSE.
        RES=.FALSE.
        IF(PTYPE.EQ.1) THEN                                   !Long downstream
        NORM=.TRUE.                                           !
        N=5                                                   !5 eqns/unks
        ENDIF                                                 !
        IF(PTYPE.EQ.2) THEN                                   !Overfall
        OVER=.TRUE.                                           !
        N=6                                                   !6 eqns/unks
        ENDIF                                                 !
```

```
      IF(PTYPE.EQ.3) THEN                                         !Res. downstream
      RES=.TRUE.                                                  !
      N=6                                                         !6 eqns/unks
      WRITE(*,303)
303   FORMAT(16X,'Give 2nd reservoir head above channel bottom')
      READ(*,*) H2                                                !Get 2nd res head
      ENDIF                                                       !
      IF(RES) WRITE(2,309) H,H2                                   !Reservoir  output
309   FORMAT(14X,'This solution is for a second reservoir downstream'/
  &   24X,'Upstream reservoir head is',1X,1F4.2/22X,'Downstream reservoi
  &   r head is',1X,1F4.2/)
      IF(NORM) WRITE(2,310) H                                     !Normal output
310   FORMAT(14X,'This solution is for a long channel downstream'/
  &   24X,'Upstream reservoir head is',1X,1F4.2/)
      IF(OVER) WRITE(2,308) H
308   FORMAT(14X,'This solution is for a free overfall downstream'/
  &   24X,'Upstream reservoir head is',1X,1F4.2/)
      WRITE(*,304)
304   FORMAT(27X,'1:  computer guesses'/27X,'2:  user guesses')
      READ (*,*) GUESS
      IF(GUESS.EQ.1) THEN
      XG(1)=20*H                                                  !Computer guesses
      XG(2)=.9*H                                                  !all depths and
      XG(3)=XG(2)                                                 !the flow
      XG(4)=1.5*YG                                                !
      XG(5)=1.5*XG(4)                                             !
      IF(RES) XG(6)=.9*H2                                         !
      IF(OVER .AND. G.GE.30) XG(6)=1.                             !
      IF(OVER .AND. G.LT.30) XG(6)=.3                             !
      GO TO 305                                                   !
      ENDIF                                                       !
      IF(NORM) THEN                                               !5 unknowns
      WRITE(*,*) '                    Guess:  Q,Y1,Yu,Yd,Yt'
      ELSE                                                        !6 unknowns
      WRITE(*,*) '                  Guess:  Q,Y1,Yu,Yd,Yt,Yend'
      ENDIF
      READ(*,*) (XG(I),I=1,N)                                     !Read guesses
305   DO 125 I=1,N
125   X(I)=XG(I)                                                  !Store in tmp array
      IF(G.GT.30.) THEN                                           !Check for si/es
      C=1.486                                                     !ES coefficient
      ELSE                                                        !
      C=1.                                                        !SI coefficient
      ENDIF                                                       !
      G2=2.*G                                                     !Convenience 2g
      KE1=1.+KE1                                                  !Convenience loss
      KL1=1.+KL1                                                  !Convenience loss
      CD1=1/CD                                                    !Convenience coeff
2     NCT=0                                                       !Set count to zero
1     DO 10 I=1,N                                                 !Outside function loop
      F(I)=FUN(I)                                                 !Call function
      DO 10 J=1,N                                                 !Inside variable loop
      DX=.005*X(J)                                                !Increment variable
```

```
      X(J)=X(J)+DX                                              !
      D(I,J)=(FUN(I)-F(I))/DX                                   !Create Jacobian matrix
10    X(J)=X(J)-DX                                              !Set back to original
      CALL SOLVEQ(N,N,6,D,F,1,DD,INDX)                          !Solve all eqns
      SUM=0.                                                    !Set error to zero
      DO 20 I=1,N                                               !Update values
      X(I)=X(I)-F(I)                                            !
20    SUM=SUM+ABS(F(I))                                         !New error
      NCT=NCT+1                                                 !Increment count
      WRITE(*,*)' NCT=',NCT,SUM                                 !Write count,error
      WRITE(*,*) (X(I),I=1,N)                                   !
      IF(NCT.LT.30 .AND. SUM.GT. ERR) GO TO 1                   !Done yet?
   !____________S O L U T I O N   F O U N D______________!
      Cc=.583+.04*Yg/X(3)                                       !Contraction coeff.
      Yj=Cc*Yg                                                  !Jet Depth
113   CALL SPACE(11)                                            !Clear screen
      IF(Yg.GT.X(3)) THEN                                       !Gate above water?
      CALL SPACE(11)                                            !Clear screen
      WRITE(*,*) "   You have specified a gate setting higher than the
   &  upstream water level.                Flow is unaffecte
   &  d by the gate."
      GO TO 124
      ENDIF
      IF(NCT.EQ.30) THEN
      IF(YG.LT..06*H) THEN
      WRITE(*,306)
306   FORMAT(28X,'Free flow conditions !!')
118   ELSE IF(RES) THEN
      IF(H2.LT..2*H) WRITE(*,*) 'The second reservoir head is too low fo
   &  r this problem to physically exist'
      IF(YG.LT..06*H) WRITE(*,306)
      ENDIF
128   WRITE(*,307)
307   FORMAT(31X,'Try another guess')
      DO 127 II=1,N
127   X(II)=XG(II)
      GO TO 124
      ENDIF
      IF(N.EQ.5) THEN
      WRITE(*,*) '   Yg     Cc    Q      Y1     Yu     Yj     Yd
   &  Yt '
      IF(NWRITE.EQ.1) THEN
      WRITE(2,*) '   Yg     Cc    Q      Y1     Yu     Yj     Yd
   &  Yt '
      WRITE(2,*) ''
      ENDIF
      WRITE(*,110) Yg,Cc,Q,Y1,Yu,Yj,Yd,Yt                       !Write results
      WRITE(2,110) Yg,Cc,Q,Y1,Yu,Yj,Yd,Yt                       !Write results
      WRITE(*,*) ''                                             !
110   FORMAT(8F8.3)                                             !
      ELSE                                                      !
      WRITE(*,*) '   Yg     Cc    Q      Y1     Yu     Yj     Yd
   &  Yt    Yend'
```

```
      IF(NWRITE.EQ.1) THEN
      WRITE(2,*) '  Yg    Cc   Q     Y1    Yu    Yj    Yd
   &  Yt   Yend'
      WRITE(2,*) ''
      ENDIF
      WRITE(*,116) Yg,Cc,Q,Y1,Yu,Yj,Yd,Yt,Yend            !Write results
      WRITE(2,116) Yg,Cc,Q,Y1,Yu,Yj,Yd,Yt,Yend            !Write results
116   FORMAT(9F8.3)
      ENDIF
      WRITE(*,*) ''
      IF(Y1.LT.YU) WRITE(*,*) 'Upstream M1 profile'
      IF(Y1.GT.YU) WRITE(*,*) 'Upstream M2 profile'
      IF(.NOT. NORM) THEN
      IF(YT.GT.Yend) WRITE(*,*) 'Downstream M2 profile'
      IF(YT.LT.Yend) WRITE(*,*) 'Downstream M1 profile'
      ENDIF
      WRITE(*,*) ''
      WRITE(*,121) NCT
      WRITE(*,122) SUM
121   FORMAT(1X,'Iterations= ',I2)
122   FORMAT(1X,'Error= ',F10.8)
      NWRITE=0
124   WRITE(*,*) ''
      WRITE(*,*) '                 Type 22 for new problem'
      WRITE(*,*) '                      -or-'
      WRITE(*,*) '                 Give new gate opening'
      CALL SPACE(9)                                        !Clear screen
111   READ(*,*) YG                                         !New gate opening
      X(4)=1.5*YG                                          !Computer guesses Yd
      FREE=.FALSE.                                         !Reinitialize free
      IF(Yg.EQ.22) GO TO 114                               !Begin new problem
      IF(Yg.GT. 0.) GO TO 2                                !Start again
      CALL SPACE(12)
      WRITE(*,*)"          Don't forget to fill out your registrati
   &  on card !"
      CALL SPACE(11)
101   STOP                                                 !Press any key....
      END                                                  !End Main Program


      FUNCTION FUN(II)
      EXTERNAL DYX
      LOGICAL*2 RES,NORM,OVER
      REAL X(6),W(1,13),KE1,KL1,Y(1),DY(1),XP(1),YP(1,1)
      COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,NORM,OVER,RES
      COMMON /TRAS/H,G,G2,Yg,C,QN,KE1,KL1,X,CD1,H2,CM
      COMMON /GEO/bg,b1,b2,So1,So2,FN1,FN2,FM1,FM2,FL1,FL2
      Q2G=X(1)*X(1)/G2
      Cc=.583+.04*Yg/X(3)
      Yj=Cc*Yg
      IF(NORM) GO TO (1,2,3,4,5),II
```

```
      IF(.NOT. NORM) GO TO (1,2,3,4,5,6),II
1     FUN=H-X(2)-Q2G/(B1*X(2)+FM1*X(2)**2.)**2.            !Entrance energy
      RETURN                                !
2     FUN=X(3)-X(4)+Q2G/((CD1*(B1*X(3)+FM1*X(3)            !Submerged energy
   &  )**2.))**2.-Q2G/(CD1*BG*YJ)**2.
      RETURN
3     FUN=.5*B2*(X(4)**2.-(CM*X(5))**2.)+                  !Submerged momentum
   &  .3333333*FM2*(X(4)**3.-(CM*X(5))**3.)+
   &  2*Q2G*(1./(BG*YJ)-1./(B2*(CM*X(5))+FM2
   &  *(CM*X(5))**2.))
      RETURN
4     Y(1)=X(3)                                            !Upstream GVF
      XX=FL1                                               !Start at gate
      XZ=0                                                 !End at reservoir
      GO TO 7                                              !Begin GVF calc's
5     IF(NORM) THEN                                        !Norm
      A=B2*X(5)+FM2*X(5)**2.                               !Norm Area
      P=B2+2*X(5)*SQRT(1.+FM2**2.)                         !Norm Perimeter
      FUN=FN2*X(1)-C*A*((A/P)**.6666666)*SQRT(SO2)         !Norm Manning's eqn.
      RETURN                                               !Skip GVF calc's
      ENDIF
      IF(RES) Y(1)=X(6)                                    !Start at Yr
      IF(OVER) Y(1)=X(6)+.1                                !Start at Yc
      XX=FL2                                               !Start at very end
      XZ=0.                                                !End at gate
7     TOL=.0001                                            !ODESOL parameters
      H1=.1                                                !
      HMIN=.001                                            !
      CALL ODESOL(Y,DY,1,XX,XZ,TOL,H1,HMIN,1,XP,YP,W,DYX)
      IF(II.EQ.4) THEN                                     !Upstream GVF
      FUN=X(2)-Y(1)                                        !Match Y1
      ELSE                                                 !Downstream GVF
      FUN=X(5)-Y(1)                                        !Match Yt
      ENDIF                                                !
      RETURN                                               !
6     IF(RES) FUN=H2-X(6)-Q2G/(B2*X(6)+FM2*X(6)            !Chanl. to res2 energy
    & **2.)**2.
      IF(OVER) FUN=G*(B2*X(6)+FM2*X(6)**2.)**3.-           !Critcal flow eqn
    & X(1)*X(1)*(B2+2*FM2*X(6))
      RETURN
      END
      SUBROUTINE DYX(XX,Y,DY)
      REAL Y(1),DY(1),KE1,X(6)
      COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,NORM,OVER,RES
      COMMON /TRAS/H,G,G2,Yg,C,QN,KE1,KL1,X,CD1,H2,CM
      COMMON /GEO/bg,b1,b2,So1,So2,FN1,FN2,FM1,FM2,FL1,FL2
      Q2G=X(1)*X(1)/G                                      !QQ/2g
      b=b1                                                 !Upstream GVF
      So=So1                                               !
      FN=FN1                                               !
      FM=FM1                                               !
      IF(XX.GT.FL1)THEN                                    !Downstream GVF
      b=b2                                                 !
```

```
      So=So2                                         !
      FN=FN2                                         !
      FM=FM2                                         !
      ENDIF                                          !
      QN=(FN*X(1)/C)**2.                             !(nQ/c)^2
      YY=ABS(Y(1))                                   !Current Depth
      A=B*YY                                         !Area
      P=B+2.*YY                                      !Perimeter
      T=B                                            !Top Width
      FRS=Q2G*T/A**3.                                !Froude #
      SF=QN*(((P/A)**.6666667)/A)**2.                !Manning's eqn.
      IF(FRS.GT.1) FRS=.9                            !Limit Froude #
      DY(1)=(So-Sf)/(1.-FRS)                         !Differential eqn.
      RETURN
      END

      SUBROUTINE SPACE(NS)
      DO 112 IS=1,NS
112   WRITE(*,*) ''
      RETURN
      END
```

Subroutines SOLVEQ and ODESOL are property of Dr. Roland Jeppson, at the USU Civil Engineering department.